The effect of a diagonal control rod in a cylindrical reactor

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ADDENDUM

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Summary:

A thin cadmium rod, corresponding to a control rod in a reactor, was placed diagonally in a cylinder containing water. The change in time decay constant of the neutron flux was measured and interpreted as a change in the geometric buckling of the system. The measurements were performed for various ratios of height to radius of the cylinder, and the results were compared with calculations.

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The effect of a diagonal control rod in a cylindrical reactor.

In an earlier report\(^1\) the effect of thin control rods on the reactivity of a reactor was studied in a model experiment using the pulsed neutron source method. Thin cadmium rods were introduced into a small cylinder containing water. The difference in the time decay constant of the neutron flux with and without a rod was determined and interpreted as a change in the geometric buckling of the system. The experimental results were compared with calculations using one-group theory which is supposed to be a good approximation. The measurements agreed well with theory for rods placed centrally along the axis of the cylinder, but not for diagonal rods. Here the theoretical predictions came out 20 - 30 per cent too high for all the five rod sizes. In order to investigate this discrepancy some more experiments and calculations have been done.

Measurements.

The discrepancy between the theory and the earlier experiments seemed to be independent of the diameter of the rods, which ranged from 0.15 to 0.74 cm. Therefore, in the present experiments the rod diameter was kept constant equal to 0.50 cm and the ratio of height \(H\) to radius \(R\) of the cylinder, which in the earlier experiments was 2.08, was varied instead. Various theories deviate from each other most strongly for small values of \(H/R\), so a larger cylinder was used (diameter 18 cm) and the height of the water was varied between 4.0 and 16.5 cm.

All other details of the experimental arrangement were the same as described in the previous report. The time decay constant of the neutron flux was measured with and without the rod in position and the buckling change \(\Delta B^2\) was calculated from the formula

\[
\Delta B^2 = \frac{\Delta \lambda}{D_0 - (C - d)(2B^2 + \Delta B^2)}
\]  

(1)

Here \(\Delta \lambda\) is the change in decay constant, \(B^2\) the buckling before the insertion of the rod, \(D_0\) the diffusion constant for zero buckling, \(C\) the diffusion cooling coefficient and \(d\) a correction term\(^2\).

The results of the measurements are given in Table 1 and Fig. 1. To facilitate the comparison with theory we have divided the
measured buckling changes with those of a central axial rod with the same
diameter, as calculated from eq. (13) of reference 1. The errors are
almost completely due to the uncertainty in $\Delta \lambda$. When $H/R$ becomes
smaller the decay constants increase, and as a result the uncertainty in
$\Delta \lambda$ increases, as is apparent from the data given in the table and figure.

Table 1.

<table>
<thead>
<tr>
<th>H/R</th>
<th>Buckling ratio</th>
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<tr>
<td>0.485</td>
<td>$2.375 \pm 0.236$</td>
</tr>
<tr>
<td>0.854</td>
<td>$1.263 \pm 0.151$</td>
</tr>
<tr>
<td>1.023</td>
<td>$1.217 \pm 0.103$</td>
</tr>
<tr>
<td>1.291</td>
<td>$0.985 \pm 0.074$</td>
</tr>
<tr>
<td>1.811</td>
<td>$0.939 \pm 0.131$</td>
</tr>
<tr>
<td>*2.079</td>
<td>$0.795 \pm 0.089$</td>
</tr>
</tbody>
</table>

* Measurement with cylinder 14.5 cm in diameter as reported in reference 1.

Theory.

The theoretical predictions in the earlier work were based on the
simple assumption that the ratio of the buckling changes caused by a dia-
gonal rod and a central rod with the same diameter should equal the ratio
of the integrals of the weighting function $\phi^2$ over the rod volumes ($\phi$ is the
undisturbed neutron flux). This ratio was calculated as

$$F = \frac{4}{\pi} \sqrt{1 + \frac{4R^2}{H^2}} \int_0^{\pi/2} J_0^2 \left( \frac{4.81 x}{\pi} \right) \cos^2 x \, dx$$  \hspace{1cm} (2)

and as said above it turned out to be an overestimation of the effect of the
diagonal rods. This is also obvious from Fig. 1 where curve A represents
calculations according to this method.

We have tried to find some other simple way of calculation to get
better agreement with the measured data. The following method was used
to obtain curve B in Fig. 1.

The cylinder is divided into thin slices by means of planes perpen-
dicular to the axis. In each such slice the control rod has an elliptic cross

section area. We assume that the effect of a small elliptic rod thus obtained is the same as that of a circular rod with effective radius $a_{\text{eff}}$ according to the formula (compare with eq. 12 in the earlier work)

$$a_{\text{eff}} = \frac{p}{2\pi} e^{-\frac{\mu \ell_{\text{tr}} \cdot 2\pi}{p}}$$

(3)

where $p$ is the circumference of the elliptic cross section, and $\mu \ell_{\text{tr}}$ is the extrapolation length for a cylinder with radius $\frac{p}{2\pi}$ in a medium with transport mean free path $\ell_{\text{tr}}$. The effect of this circular rod positioned along the axis of the cylinder is calculated from eq. (13) of reference 1. The total effect is obtained by adding the effects from the various thin slices properly weighted by a factor $\phi^2$. Finally, the same weighting procedure is carried through for an axial cylindrical rod with the same physical radius, and the ratio of the two values is calculated.

As seen from the figure this method of calculation gives a curve which lies somewhat lower than the experimental points. Thus, the two calculation methods seem to give upper and lower limits for the effect of diagonal control rods. In view of the rather arbitrary assumptions made it would be of interest to compare the experimental values with a more refined theory.

Acknowledgements.

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References.

2) N. G. Sjöstrand, Arkiv för Fysik 15, 147 (1959)

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Fig. 1. The buckling change caused by a diagonal rod divided by that of an axial control rod as a function of the height to radius ratio of the cylinder. Curve A was calculated according to eq. (2) and curve B with the method described in connection with eq. (3). The line C is the asymptot to the expression (2) and the points represent measured values.