Improvement of Reactor Fuel Element Heat Transfer by Surface Roughness

B. Kjellström and A. E. Larsson

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IMPROVEMENT OF REACTOR FUEL ELEMENT HEAT TRANSFER
BY SURFACE ROUGHNESS

by

B Kjellström and A E Larsson

SUMMARY

In heat exchangers with a limited surface temperature such as reactor fuel elements, rough heat transfer surfaces may give lower pumping power than smooth. To obtain data for choice of the most advantageous roughness for the superheater elements in the Marviken reactor, measurements were made of heat transfer and pressure drop in an annular channel with a smooth or rough test rod in a smooth adiabatic shroud. 24 different roughness geometries were tested.

The results were transformed to rod cluster geometry by the method of W B Hall, and correlated by the friction and heat transfer similarity laws as suggested by D F Dipprey and R H Sabersky with RMS errors of 12.5 % in the friction factor and 8.1 % in the Stanton number.

The relation between the Stanton number and the friction factor could be described by a relation of the type suggested by W Nunner, with a mean error of 3.1 % and an RMS error of 11.6 %.

Application of the results to fuel element calculations is discussed, and the great gains in economy which can be obtained with rough surfaces are demonstrated by two examples.

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1. INTRODUCTION

One of several ways to better reactor economy is improvement of the efficiency of the heat transfer in the fuel elements. A promising method to achieve this is roughening of the heat transfer surfaces.

Although numerous investigations on rough surface heat transfer have been performed, not much information is available on the performance of different roughness configurations which can be used in nuclear fuel elements. To obtain data for choice of the most suitable roughness for the superheater elements in the Marviken reactor an investigation was therefore started with the particular aim of studying the effect of the shape of two-dimensional roughness on friction and heat transfer. This report is a summary of the results obtained up to now. Further details of the results can be found in three internal reports [75, 76, 77].

2. EARLIER INVESTIGATION

A considerable amount of data for heat transfer and friction for flow past roughened surface is available. The most significant investigations have been collected in table 2.1. Many of them have been discussed in the literature survey by Bhattacharyya [3].

Different opinions exist as to whether optimal roughness geometries can be found or not. Nunner [45] found that an increase in heat transfer is always followed by a given increase in pressure drop. Later, however, Burgoyne et al. [7], Dipprey and Sabersky [10], Sheriff and Gumley [59], Walker and Rapier [67] and recently Wilkie [68, 69] have supplied data which show that certain geometries give lower friction for a given increase in heat transfer.

The majority of the experiments have been performed in a round duct or in an annular channel with roughened inner surface and smooth outer surface. Only a few investigations have been made with rod clusters. The most useful of these seems to be that of Sutherland [64], made with a 7-rod bundle with a shroud designed to simulate an infinite bundle.

Data obtained in the annular geometry cannot be applied to rod cluster subchannels without separation of the shear stresses at the two walls and transformation of the heat transfer coefficient to the case of
an adiabatic zero shear surface, as will be explained later in section 3.2. Such a transformation has been made by some investigators. Unfortunately the results given by others cannot easily be transformed since the necessary data are not provided. The complete transformation according to Hall [21] has been applied by Draycott and Lawther [11], Walker and Rapier [67], Wilkie [68,69], Edwards and Sheriff [13], Sheriff and Gumley [59] and Burgoyne [7]. Grunwald [18,19] measured the velocity distribution and calculated the shear stress at the rough side, but the Stanton number and the equivalent diameter were not transformed. Simplified transformations have been applied by Blomberg [4], Malherbe [36] and Bennett and Kearsey [2]. The errors introduced by the simplifications have been investigated by Wilkie [70] and shown to be in the order of 10-20%, but with large scatter. Gargaud and Paumard [16] unfortunately included the zero shear perimeter when the equivalent diameter was calculated, which makes their transformed data invalid for comparison purposes.

Hall's transformation theory has been checked by Cowin et al. [9]. The transformed rough friction factor was found to be at worst 10% lower than the actual value. This is also confirmed by shear stress measurements made by Kjellström and Hedberg [74].

Theoretical models of rough surface heat transfer and friction have been suggested by Dipprey and Sabersky [10], Kolár [32] and Nunner [45]. These theories can be used for correlation of experimental data, but cannot be used for calculation of roughness performance without such data. Of the existing theoretical models, that of Dipprey and Sabersky [10] seems to be the most elaborate. It will be used below for correlation of the results.

Table 2.1 Investigations on rough surface performance

<table>
<thead>
<tr>
<th>Abbreviations:</th>
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<tbody>
<tr>
<td>A Air</td>
<td>WG Water - Glycol mixture</td>
</tr>
<tr>
<td>O Transformer oil</td>
<td>F Friction measurements</td>
</tr>
<tr>
<td>S Steam</td>
<td>H Heat transfer measurements</td>
</tr>
<tr>
<td>W Water</td>
<td>B-O Burnout measurements</td>
</tr>
<tr>
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<td>T Transformed</td>
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### 2.1.1 Circular channels

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<th>Medium</th>
<th>Range of Re·10⁻³</th>
<th>Roughness Shape</th>
<th>Range of h/D_h·10⁻³</th>
<th>Type of experiments</th>
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<tr>
<td>Cope</td>
<td>8</td>
<td>1941</td>
<td>W</td>
<td>2-60</td>
<td>knurls</td>
<td>11-63</td>
<td>F, H</td>
</tr>
<tr>
<td>Dipprey &amp; Sabersky</td>
<td>10</td>
<td>1963</td>
<td>W</td>
<td>10-500</td>
<td>sand rough</td>
<td>2.4-49</td>
<td>F, H</td>
</tr>
<tr>
<td>Gargaud &amp; Paumard</td>
<td>16</td>
<td>1964</td>
<td>CO₂</td>
<td>200-3500</td>
<td>rect. fins, triang. fins, SI-thread, waves, herringbone protrusions</td>
<td>4-42</td>
<td>F, H</td>
</tr>
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<td>Kolár</td>
<td>32</td>
<td>1965</td>
<td>AW</td>
<td>4-200</td>
<td>triang. thread</td>
<td>19-50</td>
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</tr>
<tr>
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<td>41</td>
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<td>W</td>
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<td>14-100</td>
<td>F</td>
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<tr>
<td>Nikuradse</td>
<td>44</td>
<td>1933</td>
<td>W</td>
<td>0.5-1000</td>
<td>sand rough</td>
<td>2-66</td>
<td>F</td>
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<tr>
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<td>1956</td>
<td>A</td>
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<td>40-80</td>
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<td>1952</td>
<td>A</td>
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<td></td>
<td>54</td>
<td>1957</td>
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<td>61</td>
<td>1957</td>
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<td>10-70</td>
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<tr>
<td>O'Sullivan</td>
<td>62</td>
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<td>W</td>
<td>1-100</td>
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<td>F</td>
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<td>Sutherland &amp; Miller</td>
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<td>1958</td>
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<td>triang. thread</td>
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### 2.1.2 Annular channel, roughened inner surface

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<td>Brauer</td>
<td>5</td>
<td>1961</td>
<td>W</td>
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<td>26-136</td>
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<td>Fedynskii</td>
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<td>150-600</td>
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<td>Sheriff et al.</td>
<td>58</td>
<td>1963</td>
<td>A</td>
<td>10-100</td>
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<td>F, H</td>
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<td>Sheriff &amp; Gumley</td>
<td>59</td>
<td>1965</td>
<td>A</td>
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<td>h²=20-200</td>
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<td>60</td>
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<td>Walker &amp; Rapier</td>
<td>67</td>
<td>1963</td>
<td>A</td>
<td>200</td>
<td>rect. fins</td>
<td>4-16</td>
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<td>69</td>
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<td>A</td>
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### 2.1.3 Rectangular channels

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<th>Range of h/D_{h}·10^{3}</th>
<th>Type of experiments</th>
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<td><strong>Double sided roughness</strong></td>
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<tr>
<td>Fournel (8.0)</td>
<td>15</td>
<td>1955</td>
<td>A</td>
<td>15-60</td>
<td>wires</td>
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<td>33</td>
<td>1959</td>
<td>A</td>
<td>7-30</td>
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<td><strong>Single sided roughness</strong></td>
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<td></td>
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<td>Edwards &amp; Sheriff (12.2)</td>
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<td>1961</td>
<td>A</td>
<td>50-200</td>
<td>wires</td>
<td>31-150</td>
<td>F,H,T</td>
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<td>Malm (8.0)</td>
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<td>1960</td>
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<td>rect. fins</td>
<td>60-80</td>
<td>F,H</td>
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<td>1963</td>
<td>A</td>
<td>20-100</td>
<td>rect. fins</td>
<td>5-16</td>
<td>F,H</td>
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The figure within brackets below the name of the author refers to the aspect ratio of the channel.

### 2.1.4 Rod bundles

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<th>Roughness parameters</th>
<th>Range of h/D_{h}·10^{3}</th>
<th>Type of experiments</th>
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<td>Kidd &amp; Wantland</td>
<td>29</td>
<td>1963</td>
<td>A</td>
<td>2-200</td>
<td>W-thread wire coil</td>
<td>80</td>
<td>F</td>
</tr>
<tr>
<td>Sutherland</td>
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<td>1966</td>
<td>A</td>
<td>5-200</td>
<td>wire coil</td>
<td>7-35</td>
<td>F,H</td>
</tr>
<tr>
<td><strong>21-rod bundle</strong></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Rapier &amp; White</td>
<td>49</td>
<td>1963</td>
<td>A</td>
<td>70-220</td>
<td>square fins</td>
<td>5</td>
<td>H</td>
</tr>
</tbody>
</table>
3. THEORETICAL CONSIDERATIONS

3.1 Dimensional analysis

Present knowledge about turbulent flow is not sufficient for solution of the flow and heat transfer equations even for simple cross-sections and smooth walls. However, by application of dimensional analysis a general understanding of the problem and a good basis for correlation of experimental data can be obtained.

If the discussion is restricted to the part of the channel which is sufficiently far from the inlet to ensure that the influence of the inlet conditions is negligible, the axial pressure gradient and the heat transfer coefficient may be assumed to depend on the following parameters:

Parameters of the flow:

The mean velocity, \( \bar{u} \), the density \( \rho \), the viscosity, \( \mu \), and as far as the heat transfer coefficient is concerned the thermal conductivity \( \lambda \), and the heat capacity, \( c_p \), of the fluid. (The fluid has been assumed incompressible, which is allowable in this case since the Mach number in the experiments and the fuel elements is below 0.3, cf. Shapiro [57] and Jakob [25].)

If the fluid properties are temperature-dependent this must be considered by introduction of additional parameters. Humble [23] found that this could be avoided if all fluid properties were evaluated at the "film" temperature. We did not do this, since the experimental data could be reasonably well correlated even when the temperature dependence was neglected.

Parameters of the channel:

The shape of the cross section and a characteristic length, \( D_h \).

Parameters of the roughness:

The shape, the height, \( h \), the width at the peaks, \( b \), and the gap width, \( s-b \), equal to the pitch minus the peak width. (Only two-dimensional roughness is considered.)
Application of dimensional analysis then gives:

\[ \frac{2 \Delta p}{\rho u^2} = F_{fr} \left( \frac{u D_h \rho}{\eta}; \text{channel } \frac{h}{D_h}; \frac{\rho}{\eta}; \frac{\text{roughness}}{h}; \frac{\text{shape}}{D_h} \right) \]  

(1)

\[ \frac{\alpha}{u \rho c_p} = F_{St} \left( \frac{u D_h \rho}{\eta}; \text{channel } \frac{h}{D_h}; \frac{\rho}{\eta}; \frac{\text{roughness}}{h}; \frac{\text{shape}}{h}; \frac{c_p}{\eta} \right) \]  

(2)

The dimensionless variables which have replaced the original ones form a complete set according to the \( \pi \) theorem, cf. Langhaar [34]. Rewriting eqs. (1) and (2) with the commonly used symbols for some of the dimensionless variables and accounting for the dependence of the channel shape by a shape function \( \xi \) gives:

\[ f = \xi_{fr} F_{fr} (Re; \frac{h}{D_h}; \frac{s-b}{h}; \frac{b}{h}) \]  

(3)

\[ St = \xi_{St} F_{St} (Re; \frac{h}{D_h}; \frac{s-b}{h}; \frac{b}{h}; Pr) \]  

(4)

The characteristic length \( D_h \) will be calculated below as

\[ D_h = \frac{4A}{U} \]  

(5)

It has earlier been assumed that this makes it possible to eliminate the channel shape function, but the results of Gunn and Darling [20] indicate that this is not generally true.

It is evident that a unique description of the roughness could have also been obtained if the pitch, \( s \), instead of the gap width, \( s-b \), had been used in the third dimensionless ratio. However, it can be expected that the interaction between adjacent roughness elements is governed mainly by the gap between the protrusions. This was the reason why \( s-b \) was chosen.

Eqs. (3) and (4) makes it possible to run the experiments under different conditions from those prevailing in the channel to which the data are to be applied. The only requirement is that the dimensionless parameters are equal.
It was therefore possible to run the experiments in a large scale version, which simplified the production of the roughened surfaces. But the experimental geometry, a partially rough annulus with a rough rod in a smooth shroud, is definitely far from similar to the geometry of a reactor fuel element. The transformation of the data to fuel element geometry will be discussed in the next section.

A further discussion of the form of the functions $F_{fr}$ and $F_{St}$ can be found in section 3.3 and 3.4.

3.2 Transformation of the data obtained in an annulus to cluster geometry

3.2.1 Similarity between subchannels in clusters and annuli

In an infinite uniformly heated rod cluster it is possible to find around each rod a section across which no transfer of heat or momentum occurs. Such a cluster can therefore be treated as composed of single rods surrounded by subchannels separated by adiabatic zero section. In clusters which are not extremely close pitched these subchannels will be approximately similar in shape to the inner subchannel of an annulus between the inner surface and the zero shear section.

Friction factors determined for the inner subchannel of an annulus should therefore also be valid for subchannels in infinite clusters.

Calculation of the friction factor for the inner subchannel can be made by a method proposed by Hall [21], as will be described briefly below.

Unfortunately the thermal boundary conditions are not similar at the zero shear section of a cluster subchannel and the inner subchannel of an annulus with adiabatic outer wall. The Stanton number determined for the latter can therefore not be used for a cluster subchannel. It is possible though, as suggested by Hall [21], to calculate from experimental data for an annulus with adiabatic outer wall the Stanton number which would have been obtained if the zero shear section had been adiabatic. The procedure will be described briefly below.

In many actual fuel elements the division of the flow into annular-like subchannels around each rod and neglect of the heat and momen-
turn transfer between the subchannels will be a very rough approxima-
tion, and a more detailed analysis is necessary. The data transformed
to infinite cluster geometry will however still be applicable if sub-
channels consisting of sectors of annuli are employed and a correction
for heat and momentum transfer between the sectors is applied after-
wards as suggested by Rapier and White [49].

The effect of the slight difference in shape between the inner
subchannel of the annulus and a cluster subchannel and of the neglect
of heat and momentum transfer between subchannels must be examined
experimentally. However, it is believed that the effect is small enough
to make the results of a comparison between different roughnesses of
the inner subchannel of an annulus valid also for the cluster case.

3.2.2 Calculation of the friction factor for the inner subchannel

The method suggested by Hall [21] is used. According to
the definition of the friction factor (cf. eqs. (1) and (3)), the friction
factor for the inner subchannel will be

\[ f_1 = - \frac{2D_{h1}}{\rho \frac{\partial \rho}{\partial x} \frac{u}{u_1} u_1 } \]  \hspace{1cm} (6)

which, since \( \frac{\partial \rho}{\partial x} \) is uniform across the channel in fully developed flow,
can be written as

\[ f_1 = \frac{D_{h1}}{D_{h}} \cdot \frac{\rho u^2}{\rho_1 u_1^2} \cdot f \]  \hspace{1cm} (7)

Defining the characteristic length as four times the flow area
divided by the shear perimeter (eq. 5), and assuming the radius of
zero shear equal to the radius of maximum velocity, \( \hat{r} \), (which is only
true as an approximation, cf. Kjellström and Hedberg [74]) gives:

\[ f_1 = \frac{\hat{r}^2 - r_1^2}{r_2^2 - r_1^2} \cdot \frac{r_1 + r_2}{r_1^2 - r_1 \frac{\rho u^2}{\rho_1 u_1^2}} \cdot f \]  \hspace{1cm} (8)
This makes it possible after measurement of the axial pressure gradient and mapping of the temperature and velocity distributions to calculate the friction factor for the inner subchannel.

3.2.3 Calculation of the Stanton number for the inner channel which would have been obtained with an adiabatic zero shear surface

The method suggested by Hall [21] is used.

The governing equation for the heat transport within the fluid is:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \Lambda \frac{\partial \theta}{\partial r} \right] = \rho c_p u \frac{\partial \theta}{\partial x} \]  

(9)

where \( \Lambda \) is the equivalent turbulent thermal conductivity of the fluid. If the surface heat flux is uniform, \( \frac{\partial \theta}{\partial x} \) will be constant some distance from the inlet, and eq. (9) can be integrated to give:

\[ - \Lambda \frac{\partial \theta}{\partial r} = \frac{1}{p} \frac{\partial \theta}{\partial x} \int_r^{r_2} \rho ur dr \]  

(10)

in the experimental case where the outer surface was adiabatic.

The hypothetical temperature distribution which would have been obtained if the zero shear surface had been adiabatic must also satisfy eq. (9). The axial temperature gradient will be different, however, since in this case all the heat is absorbed by the inner channel. Since at the inner wall the temperature gradients are equal in the experimental and hypothetic distributions, if the heat fluxes are postulated equal eq. (10) together with the corresponding relation for the hypothetic distribution gives:

\[ \frac{\partial \theta}{\partial x} = \frac{\int_r^{r_2} \rho ur dr}{\int_r^{r_1} \rho ur dr} \]  

(11)

where \( \theta_1 \) represents the hypothetical temperature difference.

If it is assumed that the equivalent turbulent thermal conductivity is a property of the flow and independent of the temperature distribu-
tion (which cannot be strictly true) the radial temperature gradients are related by

$$\frac{\partial \theta}{\partial r} = \frac{\partial \theta}{\partial x} \frac{\int r \rho \text{d}r}{r} \left[ 1 - \frac{r^2}{r_2^2} \right]$$

(12)

If the temperature and velocity distributions in the experiment are known, eq. (12) may be integrated to give the hypothetic temperature distribution. The constant of integration may be chosen for instance so that the bulk temperatures of both systems are equal.

By definition the heat transfer coefficients in the two systems are

$$\alpha = \frac{\dot{q}'}{\dot{q}} \quad \alpha_1 = \frac{\dot{q}''}{\dot{q}_1}$$

(13)

and the Stanton numbers:

$$St = \frac{\alpha}{\rho c_p u} \quad St_1 = \frac{\alpha_1}{\rho_1 c_p u_1}$$

(14)

Since the heat fluxes are equal in both systems, a relation between the experimental Stanton number and that which would have been obtained if the zero shear surface had been adiabatic can be found:

$$St_1 = St \cdot \frac{\int r \rho \text{d}r}{\int r_1 \rho_1 \text{d}r} \cdot \frac{\rho u}{\rho_1 u_1}$$

(15)

The mean temperature differences are easily calculated when the distributions of the experimental and hypothetic temperatures are known:

$$\overline{\theta} = \frac{2}{(r_2^2 - r_1^2) \rho u} \int_{r_1}^{r_2} \theta \rho \text{d}r$$

(16)

$$\overline{\theta}_1 = \frac{2}{(r_2^2 - r_1^2) \rho_1 u_1} \int_{r_1}^{r} \theta_1 \rho \text{d}r$$

(17)
3.2.4 Calculation of the Reynolds number for the inner subchannel

From the definition of the Reynolds number one obtains for the inner subchannel of an annulus:

\[ \text{Re}_1 = \frac{2(r^2 - r_1^2)}{r_1} \cdot \frac{\rho_1 u_1}{\eta_1} \]  

(18)

3.3 The similarity law for friction in rough channels

It was found that the velocity distribution in the inner subchannel of the experimental annulus could be fairly well represented by

\[ u = u^+ - \frac{1}{\kappa} \ln \frac{r-r_1}{x} + \frac{\kappa_0}{2} K_1 \]  

(19)

where

\[ K_1 = \frac{r_2-r_1}{2(r-r_1)^2} \int \frac{\hat{r} \sqrt{\frac{r_1}{r} \frac{r^2}{x^2} + \frac{\kappa_0}{2} \frac{(r_2-r_1)(r-r_1)}{(r-r_1)^2}}}{x} \]  

(20)

with the constants \( \kappa_0 \) and \( \kappa \) equal to 0.14 and 0.4 respectively, cf. section 11.

The mean velocity in the inner subchannel can be obtained from eq. (19) as:

\[ \bar{u_1} = \sqrt{\frac{r_1}{\rho}} (\hat{u}^+ - B) \]  

(21)

where

\[ B = \frac{2}{r_2-r_1} \left[ \frac{1}{4\kappa} (r^2 + 2r_1 - 3r_1^2) - \frac{\kappa_0}{2} \int_{r_1}^{r} K_1 r \, dr \right] \]  

(22)

Very close to the rough surface the flow will obviously be dependent on the axial position. It will be assumed that in this region eq. (19) is still valid for the average flow. If the average velocity at the peaks of the rough surface is \( u_h \) the dimensionless maximum velocity will be obtained as:
It may be assumed that $u_h$ for a particular roughness shape is a function of the roughness height, the average shear stress at the wall, the density and the viscosity of the fluid. Application of dimensional analysis then gives:

$$u_h \sqrt{\frac{\rho}{\tau}} = f(\frac{h}{\nu})$$  (24)

A recent visual investigation by Townes and Sabersky [66] has confirmed that the flow pattern close to roughness elements is determined mainly by the dimensionless roughness height $h^+$. Since

$$\frac{u}{\sqrt{\frac{\rho}{\tau}}} = f(\frac{h}{\nu})$$  (25)

combination of eqs. (21), (23), (24) and (25) gives

$$\sqrt{\frac{g}{\kappa}} = \frac{1}{\kappa} \ln \left(\frac{h}{\nu} \frac{\tau}{\rho}\right) + f(h^+) - B \frac{1}{\nu} \cdot (K_1)_h$$  (26)

which is the friction similarity law for the inner subchannel of a rough annulus. $f(h^+)$ should have the same value for roughnesses of similar shape, independent of the magnitude of the roughness. Eq. (26) shows great similarity with the corresponding relation for rough circular channels, cf. Schlichting [56]. For such channels it has been found that the function $f(h^+)$ is constant above a critical value of $h^+$. It can be expected that the same is true for rough annuli. This will be tested by the experimental data in section 10.

### 3.4 Heat transfer from rough surfaces

#### 3.4.1 General relation

The basic equations for momentum and heat transfer in the turbulent core of the fluid read:

$$- \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \varepsilon M \frac{\partial u}{\partial r} \right) = 0$$  (27)
\[-u \frac{\partial \theta}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \varepsilon_H \frac{\partial \theta}{\partial r}) = 0 \]  

(28)

if the eddy diffusivity concept is used.

Integrating these equations from \( r \) to \( \hat{r} \) and utilizing the relations

\[ \tau_1 = -\frac{\partial P}{\partial x} \frac{r^2 - r_1^2}{2r_1} \]  

(29)

\[ \dot{q}'' = -\rho \varepsilon_H \frac{1}{r_1} \int_{r_1}^{\hat{r}} urdr \]  

(30)

we get after rearrangement

\[ \frac{\partial u}{\partial r} = \frac{1}{\varepsilon_H} \frac{\tau_1}{\rho r} \frac{r_1^2 - r^2}{r^2 - r_1^2} \]  

(31)

\[ \frac{\partial \theta}{\partial r} = \frac{1}{\varepsilon_H} \frac{\dot{q}''}{\rho c_p} \frac{r_1}{r} \int_{r_1}^{\hat{r}} urdr \]  

(32)

The last factor in eq. (32) can be written:

\[ \int_{r}^{\hat{r}} \frac{urdr}{r} = \zeta(r) \frac{r_1^2 - r^2}{r^2 - r_1^2} \]  

(33)

where \( \zeta(r) \) is a function which is greater than unity if \( u \) increases from \( r_1 \) to \( \hat{r} \). It is often assumed that \( \zeta(r) \) is a constant equal to unity, cf. Dipprey and Sabersky [10], but this is strictly true only if the velocity is constant.

It is assumed that the eddy diffusivities for momentum and heat are equal in the turbulent core of the fluid, say at distances from the wall greater than \( \delta_T \). This assumption was checked by reference to the experimental velocity and temperature distribution and found to be reasonable, cf. section 11.

The assumption made by Dipprey and Sabersky [10] that the velocity is equal to the mean velocity and the temperature difference is equal
to the mean temperature difference at the same radial position was also found to be reasonable. Eqs. (31) and (32) can then be integrated from \( r_j + \delta_T \) to the radius \( r_b \), where \( u = \overline{u}, \vartheta = \overline{\vartheta} \). If eq. (33) is utilized, one obtains:

\[
\frac{\overline{u} - u_T}{\overline{\vartheta} - \vartheta_T} = \frac{1}{r_j} \frac{c_p}{\zeta} \cdot \frac{\dot{q}''}{\zeta(r_b)}
\]

(34)

where \( u_T \) and \( \vartheta_T \) are respectively the velocity and the temperature difference at \( r = r_j + \delta_T \).

The heat flux can be written as:

\[
\dot{q}'' = \alpha \cdot \overline{\vartheta} = \alpha [\vartheta_T + (\overline{\vartheta} - \vartheta_T)]
\]

(35)

and the shear stress as

\[
\tau_j = \rho \frac{1}{2} \frac{f_1}{8}
\]

(36)

By combination of eqs. (34-36) the Stanton number is obtained as

\[
St_j = \frac{f_1 / 8}{\zeta(r_b) (1 - \frac{u_T}{u}) + \frac{u \rho c_p}{\dot{q}''} \vartheta_T \cdot \frac{f_1}{8}}
\]

(37)

It will be demonstrated below how this relation by additional assumptions can be put in special forms equivalent to the relation given by Nunner [45] and the heat transfer similarity law of Dipprey and Sabersky [10].

3.4.2 The theoretical model of Nunner [45]

3.4.2.1 Additional assumptions - Nunner [45] in his classical work on rough surface heat transfer made the following additional assumptions.

a) The distance \( \delta_T \) is equal to the thickness of the laminar sublayer \( \delta \), which is the same for velocity and temperature.

b) The wall shear stress is equal to that which would have been obtained in a smooth channel at the same Reynolds number. The additional shear caused by the roughness acts at the outer border of the laminar sublayer.

c) \( \zeta(r_b) = 1 \).
3.4.2.2 Specialization of the general equation to Nunner's [45] relation - The additional assumptions give:

\[ u_T = u_\delta = \frac{\delta \cdot \tau_s}{\eta} = \rho u^2 \cdot \frac{s \cdot \delta}{\eta} \]  

(38)

\[ \theta_T = \theta_\delta = \frac{\delta \cdot \dot{q}''}{\lambda} \]  

(39)

Elimination of \( \delta \) and insertion of \( \theta_T \) in eq. (37) gives:

\[ St_1 = \frac{f_1/s}{\zeta(r_b) + \frac{u_\delta}{u} \left[ Pr \cdot \frac{f_1}{f_s} - \zeta(r_b) \right]} \]  

(40)

which if \( \zeta(r_b) = 1 \) immediately reduces to Nunner's relation. For a smooth surface where \( f_1 = f_s \) one obtains:

\[ St_s = \frac{f_s/s}{\zeta(r_b) + \frac{u_\delta}{u} \left[ Pr - \zeta(r_b) \right]} \]  

(41)

which, if \( \zeta(r_b) = 1 \), immediately reduces to the well known relation of Prandtl, cf. for instance Jakob [24].

3.4.3 The heat transfer similarity law of Dipprey and Sabersky [10]

3.4.3.1 Additional assumptions - Dipprey and Sabersky [10] assumed that:

a) The average velocity at \( \delta_T, u_T \), is for a particular roughness configuration determined by \( \delta_T \), the roughness height \( h \), the wall shear stress \( \tau_1 \), the density \( \rho \) and the viscosity \( \eta \) of the fluid.

b) The temperature difference is in addition determined by the heat flux, \( \dot{q}'' \), the thermal conductivity \( \lambda \) and the specific heat \( c_p \) of the fluid.

c) \( \delta_T = h \) in the fully rough flow regime.

d) \( \zeta(r_b) = 1 \).
3.4.3.2 Specialization of the general equation to the heat transfer similarity law - By dimensional analysis it is found that:

\[
\frac{u_T}{\sqrt{T}} = \frac{U}{\sqrt{T}} = f\left(\frac{h}{V} \sqrt{\frac{T}{\rho}}; \frac{\delta_T}{\sqrt{\frac{1}{V}}} \right) = f(h^+; \delta_T^+) \tag{42}
\]

It is also found that

\[
\frac{\rho c_p}{\bar{q}''} \frac{\tau}{\sqrt{T} \rho} = g(h^+, Pr, \delta_T^+) \tag{43}
\]

which can be written

\[
\frac{\rho c_p}{\bar{q}''} \frac{\tau}{\sqrt{T} \rho} \cdot \sqrt{\frac{1}{\rho}} = g(h^+, Pr, \delta_T^+) \tag{44}
\]

Since \(\delta_T^+\) is an arbitrary constant number it can be omitted from the function. Combination of eqs. (25), (42) and (44) with eq. (37) then gives

\[
St = \frac{f_{1/8}}{\zeta(r_b) + \sqrt{\frac{1}{8} \left[ g(h^+, Pr) - \zeta(r_b) f(h^+) \right]}} \tag{45}
\]

In the fully rough flow regime where \(\delta_T\) can be assumed equal to \(h\), \(f(h^+)\) will be identical to the function \(f(h^+)\) in the friction similarity law. By rearrangement of eq. (45)

\[
\frac{f_{1/8} St_1}{\sqrt{f_{1/8}}} - 1 = g(h^+, Pr) - f(h^+) + \left[ \zeta(r_b) - 1 \right] \sqrt{\frac{8}{f_{1/8}} - f(h^+)} \tag{46}
\]

This can be written as

\[
\frac{f_{1/8} St_1}{\sqrt{f_{1/8}}} - 1 = g(h^+, Pr, f_{1/8}) - f(h^+) \tag{47}
\]
It was found that over the range of $h^+$ and $l$ investigated, a sufficiently accurate correlation of the function $g$ could be obtained if the dependence of the friction factor was neglected, cf. section 10.

This gives the same result as if $\zeta(r_b)$ is assumed equal to unity, i.e. the heat transfer similarity law of Dipprey and Sabersky [10]:

$$\frac{f}{8 St} - 1 = g(h^+, Pr) - f(h^+)$$

(48)

3.4.3.3 The function $g(h^+, Pr) - $ Dipprey and Sabersky [10] assumed that the flow between the roughness elements consisted of small vortices and that the Stanton number for any of the short boundary layers in the cavity between the roughness elements could be expressed approximately by a relation of the type

$$St_{vi} = C_{St} (Re_{vi})^{-p} Pr^{-q}$$

(49)

where subscript $vi$ refers to the $i$:th vortex in the cavity. By combining the effects of the different boundary layer segments they finally for the fully rough flow regime where $f(h^+)$ is independent of $h^+$ arrived at a relation for $g(h^+, Pr)$

$$g_{FR} = G_{FR} (h^+)^p Pr^q$$

(50)

where $G_{FR}$ should have the same value for all roughness of similar shape and $p$ and $q$ should be universal constants. From their own experiments with sand-roughness $G_{FR}$ was found to be 5.19, $p$ 0.20 and $q$ 0.44. It is to be expected that the last two values should also be valid for fin-type roughness in annular geometry. This will be checked by the experimental data in section 10.
4. EXPERIMENTAL EQUIPMENT AND PROCEDURE

4.1 The test rig

A general view of the equipment is given in fig. 1. Air was drawn from the outside of the building through a filter by the compressor, and was blown through the rectifier, the flow rate meter and the test section followed by a mixing chamber, to be finally released outside the building.

The test section consisted of a vertical smooth tube diam. 130.05 mm, length 3890 mm. In this, the roughened test rods were concentrically located, supported at inlet and outlet by the flanges for electric connection and at the middle by three cylindrical (diam. 3 mm) radial struts with insulated ends. Bellows at the inlet were used for compensation of differences in thermal expansion of the test rod and the shroud.

The test rods (fig. 2) were in two halves, each a steel tube, diam. 55/58 mm, length 1985 mm. These were inserted from opposite ends of the test section and joined by a bolt. The rods were heated by direct current through the tube wall. The roughness geometries tested are described in section 4.3. The eccentricity of the rod could be measured by traversable probes. The observed eccentricities are reported in appendix 1.

4.2 Instrumentation

Stagnation temperature, and static and stagnation pressures of the air were measured before the test section and in the mixing chamber at the outlet.

Three measuring stations were provided in the test section, I at 2370, II at 2870 and III at 3370 mm from the inlet. At these the static pressure was measured with 4 carefully drilled holes (diam. 0.7 mm) in the shroud tube, and a traversable probe for measurement of stagnation pressure and temperature (fig. 3) was provided for mapping of velocity and temperature distributions. At the last measuring station three fixed stagnation pressure and temperature probes were included to give circumferential distributions of velocity and tem-
temperature. During the latest experiments (rods A, A, A) the second station was provided with two diametrically opposite traversable probes.

The temperature of the test rod was measured at 2070, 2370, 2670, 2870, 3070, 3370 and 3670 mm from the inlet by thermocouples set up in pairs in copper rings which were placed into grooves in ceramic cylinders. These cylinders were fitted into the test tube. The copper rings had a clearance of 0.5 mm from the tube to prevent interference from the direct current used for heating the rod, see fig. 1.

The temperature of the shroud tube was measured at 2870 mm from the inlet by four thermocouples.

Temperatures were measured with Chromel-Alumel thermocouples, Honeywell 9B2B7, pressures with Statham pressure transducers, and the flow rate by a calibrated Foster flow meter. All data were measured by a four figure digital voltmeter and recorded on punched tape.

**4.3 Tested roughness geometries**

One smooth and 24 roughened rods were tested. Only two-dimensional "fin-type" roughnesses were considered, since manufacturing of three-dimensional roughness for practical applications would be difficult.

The work was concentrated on rectangular fins.

It was suspected that the fin width would be significant for the performance of such fins, and the main part of the experiments were run with fins which were similar with regard to this. Unfortunately the parameter s/b was chosen as constant, instead of the probably more significant parameter b/h.

The fins are of course still similar, but the observations were made in an inclined plane in the h/D, b/h, \( \frac{s-b}{h} \) space instead of in a plane perpendicular to the b/h axis.

To investigate the effect of fin width, four rods with rectangular fins with other s/b ratios were tested.

Experiments were also made with sine-shaped fins (one rod), sawtooth shaped fins (two rods) and fins with a rounded bottom (one rod).

The dimensions and shapes of the fins were checked using a shadow projector (SIP-projector, kindly placed at our disposal by
AB Stal-Laval, Nacka) where magnifications up to 50 times could be obtained. A transparent paper was placed on the ground glass plate and shadowgraphs were taken along two opposite sides of the test rod at every 100 mm from 500 to 1500 mm from the outlet end. The magnification was chosen to give at least three fins on each shadowgraph. The dimensions were then measured from the drawings. Mean values for the height, the pitch to height ratio and pitch to width ratio were calculated, together with the standard deviations from the mean values.

The measured fin-dimensions are given in table 4.3.1, and representative examples of the shadowgraphs are given in figs. 4-8, where the fin shapes can be studied. These graphs, which have been reduced from the original size, were taken at 1000 mm from the outlet, which corresponds to the middle measuring station. The letters missing in table 4.3.1 refer to roughness configurations for which experiments are or were planned and no results are yet available.

Table 4.3.1 Tested roughness geometries

<table>
<thead>
<tr>
<th>Test rod</th>
<th>Roughness shape</th>
<th>Fig</th>
<th>$\bar{h}$ mm</th>
<th>$\sigma_h$ mm</th>
<th>$s-b$ h</th>
<th>$\sigma_{s-b}$ h</th>
<th>$b$ h</th>
<th>$\sigma_b$ h</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>rectangular fins</td>
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<td>0.045</td>
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<td>0.82</td>
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<td>sine-shaped fins</td>
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<td>0.013</td>
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contd.
table contd.

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<th>Test rod</th>
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<th>Fig</th>
<th>( \overline{s} ) mm</th>
<th>( \sigma_h ) mm</th>
<th>( \frac{s-b}{h} )</th>
<th>( \frac{\sigma_{s-b}}{h} )</th>
<th>( \frac{b}{h} )</th>
<th>( \frac{\sigma_b}{h} )</th>
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<td>3.27</td>
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<td>0.060</td>
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<tr>
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<td>-</td>
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<td>0.056</td>
<td>14.97</td>
<td>0.99</td>
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<td>-</td>
</tr>
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<td>0.13</td>
<td>0.51</td>
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</tr>
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<tr>
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<td>&quot;</td>
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<td>1.77</td>
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<td>0.088</td>
<td>4.98</td>
<td>0.65</td>
<td>2.94</td>
<td>0.27</td>
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</table>

4.4 Experimental procedure

The pressure transducers were carefully calibrated against Betz micromanometers (up to 400 mm H\(_2\)O) or a mercury manometer (above 400 mm H\(_2\)O) before each run.

After a change in mass flow or electric power at least 45 minutes were allowed for conditions to stabilize. This was also checked by temperature measurements.

For each run all temperatures, pressures etc. were measured in sequence four times. Velocity and temperature traverses were made between the second and third sequence.

At least three adiabatic and three heat transfer runs were made with each rod, at Reynolds numbers corresponding to maximum and minimum and one intermediate flow rate.
5. EVALUATION OF THE DATA

5.1 The fluid properties

The properties of air were calculated either by the equations given by NBS [77] or by equations fitted to the data given there, cf. [72]. The difference between the recommended and calculated values are, for the latter cases, less than 0.2 o/oo for the density and less than 0.1 o/oo for the enthalpy.

The properties were evaluated at static pressure and temperature.

5.2 Calculation of mean values

Mean values for velocity, temperature, density and viscosity for the flow in

a) the entire channel \( (r_i = r_1; r_j = r_2) \)

b) the inner subchannel \( (r_i = r_1; r_j = r) \)

were calculated from

\[
\bar{u} = \frac{2}{r_j^2 - r_i^2} \int_{r_i}^{r_j} \rho u \, dr
\]

\[
\bar{\rho} = \frac{r_j}{r_j - r_i} \int_{r_i}^{r_j} \rho \, dr
\]

\[
\bar{T} = \frac{2}{(r_j^2 - r_i^2) \bar{u} \bar{\rho}} \int_{r_i}^{r_j} \rho u \, dr
\]

\[
\bar{\Pi} = \frac{2}{(r_j^2 - r_i^2) \bar{u} \bar{\rho}} \int_{r_i}^{r_j} \rho \, dr
\]

using the measured velocity and temperature distributions. The trapezoidal rule was used to accomplish the integrations.
5.3 Calculation of the data for the entire annulus

5.3.1 The Reynolds number and friction factor

The Reynolds number was calculated as

$$Re = \frac{4A}{U} \frac{\rho u}{\nu}$$

(55)

where the flow area and the perimeter were calculated without regard to the roughness, using the root radius of the test rod.

The friction pressure gradient was calculated by deduction of the acceleration pressure drop from the measured pressure difference between measuring stations I and III.

$$-\frac{\partial p}{\partial x} = \frac{1}{L} \left[ \Delta p_{msd} - \frac{\dot{m}}{A} (u_{III} - u_I) \right]$$

(56)

and the friction factor then from

$$f = -\frac{4A}{U} \frac{\partial p}{\partial x} \cdot \frac{2}{\rho u^2}$$

(57)

5.3.2 The Stanton number

5.3.2.1 Calculation of necessary temperatures - The surface temperature of the test rod was calculated from the measured value by deduction of the temperature drop through the wall, which was small, of the order of 0.5 °C.

The bulk temperature of the air at some axial position was calculated from the measured inlet temperature and the integrated heat input up to the position considered, assuming a perfect heat balance.

The adiabatic wall temperature was calculated from

$$T_{aw} = T_t - (1 - \gamma_{aw}) \cdot \frac{-u}{2c_p}$$

(58)

where, according to Jakob [25] pp. 452 and 490,

$$\gamma_{aw} = \sqrt[3]{Pr} = 0.89$$

(59)

Calculation of temperatures along the outer wall was made on the assumption of a linear temperature increase with the same gradient as the overall value for the bulk flow, and using the measured value at 2870 mm from the inlet.
5.3.2.2 Calculation of the local heat flux - The local heat generation was calculated using the local resistance, which was obtained from the measured resistance at room temperature and the separately measured temperature dependence of the resistivity (cf. section 7).

The effect of axial conduction in the rod was found to be negligible. The local heat flux could therefore be set equal to the heat generation.

The convection heat flux was obtained after deduction of the radiative heat flux:

\[
\dot{q}' = \dot{q}_\text{tot} - \frac{5.669 \cdot 10^{-8} \cdot \varepsilon}{U_1} \left( \frac{T_1^4}{1 + \frac{U_2}{U_1} (1 - \varepsilon)} - T_2^4 \right)
\]

where \(\varepsilon\) by separate tests had been found to be 0.8 (cf. section 7).

5.3.2.3 Heat transfer coefficient and Stanton number - The heat transfer coefficient was calculated as

\[
\alpha = \frac{\dot{q}''}{T_1 - T_\text{aw}}
\]

and the Stanton number as

\[
St = \frac{A \cdot \alpha}{\dot{m} \cdot c_p}
\]

5.4 Determination of the radius of maximum velocity

The radius of maximum velocity was determined from the experimental velocity distributions by a numerical method which has been described in detail in an internal report [76].

In principle the method consists in first finding a velocity limit such that between the outermost observations on each side of the distribution which lie above this velocity, either less than four observations lie above or more than 25% of the observations lie below the limit.

By trial and error two parabolas with a common maximum, one on each side of the maximum velocity radius, were then fitted to the ob-
served velocity points where on each side the number of observations considered was equal to the number of data between the outermost observations above the limit.

By this method it was possible to find the maximum value even for non-symmetrical distributions, only the data in the vicinity of the maximum were considered, and the number of data considered was increased if the scatter was large.

The method was tested on analytical functions with excellent results. The performance of the method when applied to the experimental velocity distributions can be studied in figs. 13 and 14.

5.5 Transformation of the data to cluster geometry

The method of Hall [21], which has been described above in section 3.2, was utilized. The mean values were calculated as shown in section 5.2. The transformed Stanton number was calculated from the experimental value determined at the second measuring station, i.e. at the same location where the traverses were made.

5.6 Calculation of the functions $f(h^*)$ and $g(h^*, Pr)$ in the friction and heat transfer similarity laws

For each test run the radius of maximum velocity, the friction factor for the inner channel and the transformed Stanton number for the inner channel were determined. Using the known roughness height (table 4.3.1) $f(h^*)$ could be calculated from eq. (26), $h^*$ from eqs. (24) and (25) and finally $g(h^*, Pr)$ from eq. (48).

6. EXPERIMENTAL ERRORS

The expected standard deviation for the different results is given in the table below. For the radius of the maximum velocity it was calculated from results of special experiments and the results of the smooth rod experiments. For the other quantities it has been based on the uncertainties of the measured data.
Radius of maximum velocity 0.9 mm

Data for the entire annulus:
- Friction factor 4.1 %
- Stanton number 6.4 %
- Reynolds number 2.2 %

Transformed data, inner subchannel:
- Equivalent diameter 0.124 
- Friction factor 6.0 %
- Stanton number 6.4 %
- Reynolds number 4.5 %

Repeat tests including dismantling and reassemblage of the test section were performed with rods A, F, J, K, P and Q. The scatter of the results was about what could be expected from the estimation above.

The uncertainties of the dimensions and shapes of the roughness elements must also be considered. Information about this can be found in table 4.3.1. It is evident that the results for test rod J is of small practical interest, owing to the large uncertainties of the roughness parameters.

One rod, B, which had the most irregular fin shape was run with flow in two opposite directions. The standard deviation from the mean value of the friction factor was found to be 8.2 % and the standard deviation of the Stanton number 7.3 %. This shows, of course, only the effect of unsymmetry in the roughness, not the effect of departure from the ideal shape.

7. INTRODUCTORY EXPERIMENTS

Several introductory experiments were performed to check the reliability of the equipment and to obtain data necessary for the evaluation of the results.

The efficiency of the mixing chamber, the velocity distribution with respect to angle in the test section and the performance of the combination probes for measurement of temperature and stagnation pres-
sure were studied. The heat losses, the surface emissivity, the resistance distribution and the temperature dependence of the resistivity for the test rods were measured.

Measurements of the friction factor for flow in the circular channel formed by the shroud tube when the test rod was removed were also performed. These measurements were repeated after the rough surface experiments.

The results are shown in fig. 9 and compared with the data of Nikuradse [43]. The agreement is excellent, with an RMS difference of 3.4%.

It should be mentioned also that the heat balance was checked for every experiment. Experiments where the error exceeded 5% were excluded from further analysis.

8. RESULTS FOR SMOOTH AND PARTIALLY ROUGH ANNULI

8.1 The smooth annulus

8.1.1 Heat transfer and friction

Examples of the axial distributions of the Stanton number are shown in fig. 10. The results show that the thermal conditions are fully established at the second measuring station ($x/D_h = 39.8$).

The total passage friction factor and Stanton number are shown in fig. 11 and compared to the predictions of Gunn and Darling [20] and Kays and Leung [27] for annuli. The agreement is quite satisfactory, though the experimental data are generally lower than the predictions.

By comparison with the relations for circular channels of Nikuradse [43]

$$\frac{1}{\sqrt{f}} = 2.0 \log Re \sqrt{f} - 0.8$$

and McAdams [39]

$$St = 0.023 Re^{-0.2} Pr^{-2/3}$$

it is found that the friction factor is higher and the Stanton number lower in an annulus with an adiabatic outer surface than in a circular channel.
8.1.2 Velocity and temperature distributions

Examples of the velocity and temperature distributions measured in the smooth annulus are shown in fig. 13. The position of the calculated maximum point is shown by a short vertical line.

8.1.3 The radius of maximum velocity

The accuracy of the transformed data is very much dependent on the accuracy of the determination of the radius of maximum velocity.

A valuable check on this can be obtained by comparison with results of earlier investigations. Kays and Leung [27] have correlated data of Barrow [1], Knudsen and Katz [31], Lorenz [35], Owen [47] and Rothfus [51]. They neglected the possible influence of the Reynolds number and obtained the equation:

$$\frac{r_1 - r_z}{r_2 - r_1} = \left(\frac{r_1}{r_2}\right)^{0.343}$$

(65)

They also estimated the uncertainty limits of the data of these authors, as shown in fig. 12. This correlation also predicts with good accuracy the later results of Brighton and Jones [6], Nicol and Medwell [42] and of Grunwald [19], as well as those of the present investigation. For the latter, the mean values for the adiabatic runs and for all runs are shown, together with the corresponding standard deviations from the mean values. These standard deviations are higher than the earlier estimates, since the effect of the Reynolds number has been neglected here. Using a t-test it can be found that the difference between the mean values and the prediction by Kays and Leung [27] is not significant even at the 0.1 level.

It is therefore concluded that the results for the maximum velocity diameter in the smooth annulus are in satisfactory agreement with earlier data.

8.2 Partially rough annuli

8.2.1 Heat transfer and friction

Examples of the axial distributions of the Stanton number are shown in fig. 10. The variations are greater than for the smooth annu-
lus and are probably caused by variations in the roughness dimensions. The full results for the partially rough annuli were given in an earlier report [75] and will not be repeated here. However, the economic consequences of the data for partially rough annuli are discussed in section 12.

8.2.2 Velocity and temperature distributions

Examples of the velocity and temperature distributions measured with test rod  are shown in fig. 14. As earlier, the position of the calculated maximum is shown by a short vertical line.

9. EXPERIMENTAL VALUES FOR THE FACTOR $\zeta(r_b)$

To be able to judge the importance of the factor $\zeta(r_b)$, which is a measure of the similarity between the shear stress and heat flux distributions, cf. eqs. (31-33), the experimental data from five smooth rod experiments and 24 rough rod experiments, chosen at random, were utilized for calculation of $\zeta(r_b)$, eq. (33).

The results are summarized in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Mean value</th>
<th>Standard deviation</th>
<th>Max value</th>
<th>Min value</th>
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</thead>
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<td>1.062</td>
<td>0.006</td>
<td>1.071</td>
<td>1.059</td>
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<tr>
<td>Rough rod</td>
<td>1.112</td>
<td>0.021</td>
<td>1.162</td>
<td>1.085</td>
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</table>

10. RESULTS TRANSFORMED TO CLUSTER GEOMETRY

10.1 Results for the smooth surface

The transformed results from the smooth rod experiments are shown in fig. 15. The agreement with the corresponding results of Wilkie [68] is encouraging.
10.2 Results for rough surfaces

10.2.1 Experimental values for $f(h^+)$ and $g(h^+, Pr)$

The experimental values for $f(h^+)$ and $g(h^+, Pr)$ for the tested surfaces are shown in figs. 16-19.

Figs. 16-18 give the results for rectangular fins with $s/b \sim 8$. The data for geometrically similar fins are compared in each of figs. 16-17.

The latter confirm the assumption of the friction and heat transfer similarity laws, namely that the functions $f(h^+)$ and $g(h^+, Pr)$ are independent of the magnitude for roughnesses of similar shape. The same conclusion was made by Dipprey and Sabersky [10] for sand-roughness in a circular channel and recently by Sheriff and Gumley [59] for wire coils in infinite cluster geometry. (Data transformed from annulus.)

The form of the functions $f(h^+)$ and $g(h^+, Pr)$ also agrees well with the findings of Dipprey and Sabersky [10], i.e. $f(h^+)$ is a constant and $g(h^+, Pr)$ varies with $h^+$, as predicted by eq. (50).

This makes it possible in the future to reduce considerably the amount of experimental work necessary in studying the performance of rough surfaces.

The scatter of the data is probably to a great extent due to the uncertainties of the roughness parameters (including the shape).

$$f(h^+) \text{ and } C_{FR}' = C_{FR} \cdot Pr^q$$ for the tested roughness geometries are given in the table below.
Table 10.2.1

\( f(h^+) \) and \( G_{FR}^* \) for different types of roughness.

<table>
<thead>
<tr>
<th>Roughness shape</th>
<th>Test rods</th>
<th>( s/b )</th>
<th>( b/h )</th>
<th>( s-b ) ( h )</th>
<th>( f(h^+) )</th>
<th>( G_{FR}^* )</th>
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<tr>
<td>Rectangular</td>
<td>X, Y</td>
<td>~8</td>
<td>0.25</td>
<td>1.6</td>
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<tr>
<td></td>
<td>C, W</td>
<td></td>
<td>0.55</td>
<td>3.4</td>
<td>3.08</td>
<td>4.15</td>
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<tr>
<td></td>
<td>B, D, F, V</td>
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<td>1.0</td>
<td>7.2</td>
<td>3.03</td>
<td>4.71</td>
</tr>
<tr>
<td></td>
<td>E, N</td>
<td></td>
<td>1.9</td>
<td>13.7</td>
<td>4.08</td>
<td>4.91</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td></td>
<td>2.3</td>
<td>16.5</td>
<td>4.72</td>
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<tr>
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<td>K</td>
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<td>9.4</td>
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<td>3.74</td>
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<tr>
<td></td>
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<td>2.9</td>
<td>20.5</td>
<td>6.97</td>
<td>4.05</td>
</tr>
<tr>
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<td>R</td>
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<td>3.3</td>
<td>24.2</td>
<td>5.97</td>
<td>4.53</td>
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<tr>
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<td></td>
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<td>11.9</td>
<td>6.44</td>
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<td>U</td>
<td></td>
<td>4.5</td>
<td>3.9</td>
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<td>6.49</td>
</tr>
<tr>
<td></td>
<td>Å</td>
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<td>1.0</td>
<td>26.8</td>
<td>6.08</td>
<td>5.22</td>
</tr>
<tr>
<td></td>
<td>Ä</td>
<td>2.7</td>
<td>2.9</td>
<td>5.0</td>
<td>7.38</td>
<td>4.31</td>
</tr>
<tr>
<td>Sawtooth</td>
<td>H₂ 1(^{)}</td>
<td>-</td>
<td>-</td>
<td>8.5</td>
<td>3.49</td>
<td>4.66</td>
</tr>
<tr>
<td></td>
<td>T₁ 2(^{)}</td>
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<td>-</td>
<td>15.0</td>
<td>8.52</td>
<td>5.30</td>
</tr>
<tr>
<td></td>
<td>T₂</td>
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<td>-</td>
<td>15.0</td>
<td>5.55</td>
<td>4.86</td>
</tr>
<tr>
<td>Sine-shaped</td>
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<td>-</td>
<td>-</td>
<td>7.9</td>
<td>4.51</td>
<td>5.18</td>
</tr>
<tr>
<td>Round bottom</td>
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<td>8.5</td>
<td>1.9</td>
<td>14.1</td>
<td>6.39</td>
<td>5.37</td>
</tr>
</tbody>
</table>

1\(^{)\} Cutting edge upstream  
2\(^{)\} Cutting edge downstream

The results for rectangular fins are further analyzed below in section 10.2.2.

10.2.2 Variation of \( f(h^+) \) and \( G_{FR}^* \) with the fin parameters for rectangular fins

Fig. 20 shows a plot of \( f(h^+) \) and \( G_{FR}^* \) for rectangular fins with \( s/b = 8 \). The data, particularly for \( G_{FR}^* \), show considerable scatter, but nevertheless an attempt was made to represent them by simple curves.
Using the readings from these curves, friction factors and Stanton numbers were calculated for the conditions prevailing in the experiments and the results compared to the experimental values. The result of the comparison is shown in figs. 23 and 24. The RMS values of the relative differences \( \frac{f_{\text{calc}} - f_{\text{exp}}}{f_{\text{exp}}} \) and \( \frac{St_{\text{calc}} - St_{\text{exp}}}{St_{\text{exp}}} \) were found to be 12, 5 and 8.1 % respectively, which is quite satisfactory if all the uncertainties of the experimental data, including the uncertainties of the dimensions and shapes of the roughnesses, are considered.

It can be seen from fig. 24 that the mean difference is not zero for the Stanton number. The reason for this is that the number of experiments behind each data point in fig. 20 was not considered when the curve was drawn, i.e. the results for each particular geometry were given the same weight.

Finally, several cross-plots were tried in order to utilize the results obtained for rectangular fins with \( s/b \neq 8 \), and to obtain a picture of the variation with \( \frac{s-b}{h} \) and \( b/h \) of \( f(h^+) \) and \( G'_{\text{FR}} \) for rectangular fins in general. The results are shown in figs. 21 and 22. Owing to the limited number of experiments outside \( s/b = 8 \), the curves must be considered as very approximate. They might be useful for planning further experiments, however.

10.2.3 Calculated friction factors and Stanton numbers for rectangular fins with \( s/b = 8 \)

Calculation of friction factors and Stanton numbers at \( Re = 5 \cdot 10^5 \) was performed for rectangular fins with \( s/b = 8 \). The result is shown in fig. 25. The friction factor has been plotted as a function of \( (s-b)/h \) for constant values of the Stanton number. It is evident that although variations occur, the friction factor is almost constant for a constant Stanton number for this type of roughness.

10.3 Comparison with the results of earlier investigators

Inspection of table 2.1 reveals that only the following investigations can be used for comparison:
Transformed data for annuli
Burgoyne et al. [7]
supersedes ref. [67]
Wilkie [68, 69]

Data for circular channels
Gargaud and Paumard [16]
Sutherland and Miller [63]

Unfortunately, Burgoyne et al. [7] did not consider the effect of the fin width, and reported data obtained by cross-plotting results for rectangular fins of varying width-to-height ratios, without stating the value of this ratio. It is not possible to work backwards from the data in the report to the experimental data. Since it has been found later that the fin width is important it is not easy to utilize their results. Comparison will therefore be made only with the other three investigations.

Wilkie [68, 69] has presented results for square fins with \( \frac{s-b}{h} = 4, 6.2 \) and 8.4. The data given in his reports have been subject to some smoothing due to the method used for statistical analysis of the experiments. The value of \( r \) for which the results are valid is not stated in the reports, but it can be calculated approximately assuming that the even values of \( h/D_h \), at which data are given, correspond to the heights of the tested roughnesses.

Utilizing figs. 21 and 22, \( f(h^+) \) and \( G_{FR} \) for these types of roughness could be estimated, and the friction factor and Stanton number then calculated.

A summary of the results is given in the table below. Fig. 28 shows the data of Wilkie [68] for \( \frac{s-b}{h} = 6.2 \) together with the calculated data for this roughness.
Table 10.3.1  Comparison with the data of Wilkie [68, 69] for square fins

<table>
<thead>
<tr>
<th>s-b ( \frac{h}{h_0} )</th>
<th>( f(h^+) = 4.0 )</th>
<th>( G'_{FR} = 3.95 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>friction factor</td>
<td>Stanton number</td>
</tr>
<tr>
<td></td>
<td>mean dev. %</td>
<td>mean dev. %</td>
</tr>
<tr>
<td>h/D_h</td>
<td>RMS dev. %</td>
<td>RMS dev. %</td>
</tr>
<tr>
<td>0.004</td>
<td>-8.5</td>
<td>-0.06</td>
</tr>
<tr>
<td>0.0056</td>
<td>-3.8</td>
<td>-4.3</td>
</tr>
<tr>
<td>0.0072</td>
<td>-2.7</td>
<td>-6.9</td>
</tr>
<tr>
<td>0.0088</td>
<td>-3.2</td>
<td>-7.9</td>
</tr>
<tr>
<td>0.0104</td>
<td>-9.8</td>
<td>-8.6</td>
</tr>
<tr>
<td>total</td>
<td>-5.6</td>
<td>-5.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>s-b ( \frac{h}{h_0} )</th>
<th>( f(h^+) = 3.1 )</th>
<th>( G'_{FR} = 4.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>friction factor</td>
<td>Stanton number</td>
</tr>
<tr>
<td></td>
<td>mean dev. %</td>
<td>mean dev. %</td>
</tr>
<tr>
<td>h/D_h</td>
<td>RMS dev. %</td>
<td>RMS dev. %</td>
</tr>
<tr>
<td>0.004</td>
<td>-5.9</td>
<td>0.2</td>
</tr>
<tr>
<td>0.0056</td>
<td>-8.9</td>
<td>-0.2</td>
</tr>
<tr>
<td>0.0072</td>
<td>-11.6</td>
<td>-0.2</td>
</tr>
<tr>
<td>0.0088</td>
<td>-11.7</td>
<td>1.4</td>
</tr>
<tr>
<td>0.0104</td>
<td>-7.9</td>
<td>3.2</td>
</tr>
<tr>
<td>total</td>
<td>-9.2</td>
<td>0.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>s-b ( \frac{h}{h_0} )</th>
<th>( f(h^+) = 3.1 )</th>
<th>( G'_{FR} = 4.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>friction factor</td>
<td>Stanton number</td>
</tr>
<tr>
<td></td>
<td>mean dev. %</td>
<td>mean dev. %</td>
</tr>
<tr>
<td>h/D_h</td>
<td>RMS dev. %</td>
<td>RMS dev. %</td>
</tr>
<tr>
<td>0.004</td>
<td>-13.9</td>
<td>1.6</td>
</tr>
<tr>
<td>0.0056</td>
<td>-9.2</td>
<td>-2.3</td>
</tr>
<tr>
<td>0.0072</td>
<td>-7.1</td>
<td>-4.9</td>
</tr>
<tr>
<td>0.0088</td>
<td>-7.8</td>
<td>-6.3</td>
</tr>
<tr>
<td>0.0104</td>
<td>-12.7</td>
<td>-7.3</td>
</tr>
<tr>
<td>total</td>
<td>-10.1</td>
<td>-3.8</td>
</tr>
</tbody>
</table>

The agreement is quite satisfactory.
Wilkie [69] has also presented data for the influence of the rib width on the friction factor at $s/h = 7.2$. His data are compared to values calculated from fig. 21 in fig. 29. The calculated curve for $s/h = 7.9$, which has direct experimental background at $b/h = 1$ and 2.9 is also included. Wilkie's [69] data are considerably higher. This is surprising, in view of the good agreement with the rest of his data, but might be explained by the fact that Wilkie [69] made these experiments in a smaller channel and used a simplified transformation method, or by the comparatively limited experimental background in both investigations.

Sutherland and Miller [63] made measurements with steam in a circular channel provided with square fins, with $\frac{s-b}{h} = 10.8$. For such fins $f(h^+)\can$ be expected to be of the order of 3.2. This gives a friction factor of 0.057, to be compared with the experimental value of about 0.038. The agreement is poor, which might be explained by the difference in channel shape or by departure of the roughness from the ideal shape and nominal dimensions.

Comparison with the data of Gargaud and Paumard [16], which were obtained with rectangular fins in a circular channel, is made in the table below. The agreement is moderate, which may be explained as above. It is unfortunately not possible to utilize their data for an annular geometry, for reasons explained in section 2.

<table>
<thead>
<tr>
<th>$h/D_h$</th>
<th>$s-b/h$</th>
<th>$b/h$</th>
<th>$f(h^+)$</th>
<th>$f_G$</th>
<th>$f_G-f_{calc}$</th>
<th>$S_{calc}$</th>
<th>$S_G$</th>
<th>$S_{G-calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>9</td>
<td>1</td>
<td>3.1</td>
<td>4.5</td>
<td>0.095</td>
<td>0.084</td>
<td>-0.117</td>
<td>0.0049</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>1</td>
<td>3.1</td>
<td>4.2</td>
<td>0.061</td>
<td>0.064</td>
<td>+0.042</td>
<td>0.0043</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>1</td>
<td>3.1</td>
<td>4.5</td>
<td>0.061</td>
<td>0.050</td>
<td>-0.186</td>
<td>0.0041</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>1</td>
<td>4.0</td>
<td>4.0</td>
<td>0.063</td>
<td>0.060</td>
<td>-0.040</td>
<td>0.0045</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>1</td>
<td>3.1</td>
<td>4.2</td>
<td>0.074</td>
<td>0.064</td>
<td>-0.134</td>
<td>0.0047</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>0.6</td>
<td>4.0</td>
<td>4.0</td>
<td>0.080</td>
<td>0.072</td>
<td>-0.094</td>
<td>0.0049</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>0.6</td>
<td>2.95</td>
<td>4.25</td>
<td>0.099</td>
<td>0.082</td>
<td>-0.173</td>
<td>0.0051</td>
</tr>
</tbody>
</table>
Of the above investigations, those of Wilkie [68, 69] must be considered the most accurate. The good agreement with his results is therefore very encouraging, and this is not cancelled by the poor agreement with the data of Sutherland and Miller [63] and the varying but not poor agreement with the results of Gargaud and Paumard [16].

10.4 Comparison with the theory of Nunner [45]

10.4.1 Modification of Nunner's relation

By combination of eqs. (40) and (41) one can obtain a relation between the relative Stanton number $St/St_s$ and the relative friction factor $f/f_s$.

Before this is utilized it is interesting to compare the results for the smooth annulus with the prediction of eq. (41). The constants $C_{St_s}$ and $n_s$ in the equation

$$St_s = C_{St_s} Re_s^{n_s}$$

were in the best representation of the experimental data found to be 0.042 and -0.235 respectively.

After combination with this relation, eq. (41) could be utilized for calculation of the friction factor. The experimental value for $\zeta(r_b)$ was used, and the same expression for $u_s/\overline{u}$ as was employed by Nunner [45] was utilized. This gave with $Pr = 0.7$

$$f_s = (0.356 - 0.194 Re^{-1/8}) Re^{-0.235}$$

Comparison of eq. (67) with experimental data has been made in fig. 15. The agreement is moderate but better than for the smooth tube relation. It seems therefore justified to use a value of $\zeta(r_b)$ different from unity for annular channels.

The relation between the relative Stanton number and relative friction factor will then be

$$\frac{St}{St_s} = \frac{\zeta(r_b)_s + 1.5 Re^{-1/8} Pr^{-1/6} [Pr - \zeta(r_b)_s]}{\zeta(r_b)_s + 1.5 Re^{-1/8} Pr^{-1/6} [Pr \frac{f}{f_s} - \zeta(r_b)_s]} \frac{f}{f_s}$$

(68)
10.4.2 Comparison with experimental data

Comparison of the experimental values with the prediction of eq. (68) has been made in fig. 30. The mean relative difference, $\frac{(St/St_s)^{calc}-(St/St_s)^{exp}}{(St/St_s)^{exp}}$, was found to be $+3.1\%$ and the RMS difference $11.6\%$. This means that the accuracy of the modified Nunner relation may be sufficient for many applications.

11. ANALYSIS OF THE VELOCITY AND TEMPERATURE DISTRIBUTIONS

11.1 Universal velocity distribution

As mentioned in section 3.3, an attempt was made to represent the experimental velocity distributions by a velocity defect relation, eq. (19). Two values of the constant $\kappa_0$ were tried, $\kappa_0 = 0$, i.e., the conventional velocity defect law and $\kappa_0 = 0.14$. A slightly better agreement was obtained with the latter value, which was chosen on the basis of the results for the turbulent mixing length obtained by Nikuradse [43], cf. [76].

Examples of comparisons between the prediction and the experiments are shown in figs. 26 and 27.

The mean square of the difference was 0.135 for the smooth rod experiments, and in total 0.176. It varied, however, for the different test rods as shown in fig. 27.

In fig. 26 data of Brighton and Jones [6] for an extreme annulus ($d_2/d_1 = 16$) have also been included. The agreement with the prediction is satisfactory and much better than with the conventional velocity defect law, even if the differences are still large close to the inner surface.

11.2 The ratio of the diffusivities for heat and momentum

Local values of the ratio between the eddy diffusivities for heat and momentum were calculated from the measured velocity and temperature distributions. The scatter was considerable, as could be expected, and concealed any effect of the position in the channel. Mean values over the entire channel were therefore computed.

The results are given in the table below:
### 11. The radii for mean velocity and mean temperature difference

In deriving the heat transfer similarity law, Dipprey and Sabersky [10] assumed that the velocity equals the mean velocity and the temperature the mean temperature at the same distance from the wall.

This was checked using the experimental velocity distributions and the transformed temperature distributions (cf. section 3.2), and was found to be a good approximation since the temperature difference at the radius of the mean velocity differed as an average only 2.7 ± 0.4 % (95 % confidence interval) from the mean temperature difference.

### 12. The advantages of rough heat transfer surfaces

#### 12.1 General

Comparison of different heat transfer surfaces must be based on an analysis of their effect on the overall economy of the plant in which they are used. It is impossible to make such an analysis general enough to make it applicable to every particular plant. It has therefore been preferred below to analyse two simple examples of application and in this way show that economic gains are possible by use of rough surfaces. It should be evident that the results of this analysis cannot in general be applied to an actual plant, and that in each actual case the special conditions prevailing must be considered.

#### 12.2 Economic considerations

The costs connected to a heat exchanger is composed of operation costs, mainly determined by the energy loss due to the pressure
drop, and of fixed charges which are mainly determined by the area of the heat transfer surface and to some extent the energy loss.

Assume that the cost functions are linear, which in any case is true within limited ranges. If the operation costs and fixed charges for the energy loss are combined one obtains the total cost as

$$E_{\text{tot}} = k_p \cdot P + k_s \cdot UL \quad (69)$$

For a particular application $k_p$ will be independent of the surface conditions whereas $k_s$ may or may not be constant. It will be assumed below however that the manufacturing costs are the same regardless of the roughness of the surface.

If the shape of the cross-section is fixed, the energy loss will be a function of the perimeter and it is possible to find the optimum solution by differentiation of eq. (69) and solving for $U$ in the equation

$$\frac{\partial E_{\text{tot}}}{\partial U} = 0 \quad (70)$$

Comparison of different surfaces can be made either if the geometry of the heat exchanger is fixed relative to the total costs calculated from eq. (69), or if the geometry is to be optimized, by comparison of the total costs for the optimum case.

12.3 Comparison of the economy of different surface roughnesses in a heat exchanger with given dimensions

In many applications the geometry of the heat exchanger is fixed. We may first see how the economy in this case can be improved by surface roughness.

12.3.1 Theoretical considerations

Since the measurements were made in an annulus with a rough rod in a smooth adiabatic shroud it is interesting to study the economic consequences of the results for such heat exchangers. In principle the analysis below is not restricted to the annular shape and the result of the theoretical considerations, eq. (82), is valid generally for heat exchangers, i.e. also reactor fuel elements, where the geometry is
fixed. As an example an annular air cooled condenser will be studied. The condensation occurs inside the central tube of the annulus and the air flows in the channel between this and the shroud. The surface temperature at the inner surface is limited by the desired steam pressure in the condenser. It is assumed that the equipment has been optimized for a smooth surface on the air side of the inner surface and that later an increase in the performance of the condenser has been found necessary.

This can be achieved either by increasing the mass flow on the air side and thus increasing the heat transfer due to the increased velocity, or by introduction of a rough heat transfer surface.

Since the dimensions of the apparatus are all fixed, the only factor which will affect the economy is the necessary pumping power on the coolant side. We examine how this is affected by the surface conditions of the heat transfer surface.

Heat balance on the coolant side gives

\[ q = \dot{m} c_p (T_u - T_1) \]  

(71)

where \( \dot{m} \) is the coolant mass flow and \( T_u \) and \( T_1 \) the outlet and inlet temperatures, the latter being constant.

The maximum surface temperature can be calculated from

\[ T_w = T_u + \frac{\dot{q}}{\pi d_1 \cdot L \cdot \alpha} \]  

(72)

The pumping power can be obtained from

\[ P = \frac{\dot{m}}{\rho} \Delta p \]  

(73)

where

\[ \Delta p = \frac{L}{d_2 - d_1} \cdot \frac{\dot{m}^2}{2 \rho} \left( \frac{4}{\pi (d_2^2 - d_1^2)^2} \right) \]  

(74)

At least in limited ranges of the Reynolds number the heat transfer coefficient and friction factor can be obtained from

\[ \alpha = \frac{4 \dot{m} c_p}{\pi (d_2^2 - d_1^2)} C_{St} \cdot Re^n \]  

(75)
\[ f = C_f \cdot \text{Re}^m \]  

(76)

where \( C_{St}, C_f, m \) and \( n \) are dependent of the surface conditions.

By combination of eqs. (71), (72) and (75) an equation is obtained from which the mass flow can be solved:

\[ \frac{\dot{q}}{\dot{m} c_p} + \frac{\dot{q} \beta_1}{C_{St} \cdot \dot{m}^{1+n} \left( \frac{\text{Re}}{\dot{m}} \right)^n} = T_w - T_i \]  

(77)

where

\[ \beta_1 = \frac{d_2^2 - d_1^2}{4 c_p d_1 L} \]  

(78)

The first term, equal to the temperature rise of the coolant, is normally small in comparison with the second, and will be neglected in the following. Note that \( \text{Re}/\dot{m} \) is independent of \( \dot{m} \).

Solving this equation for the mass flow, and combining eqs. (74) and (76) using the value found for the mass flow in this equation, gives the necessary pumping power

\[ P = \left( \frac{\dot{q} \beta_1}{T_w - T_i} \right)^{3+m \over 1+n} \cdot \beta_2 \cdot \left( \frac{\text{Re}}{\dot{m}} \right)^{m-3n \over 1+n} \cdot \frac{C_f}{C_{St}} \]  

(79)

where

\[ \beta_2 = \frac{8 L (d_2^2 - d_1^2)^2}{\pi \rho \nu^2 (d_2 - d_1)} \]  

(80)

It is interesting to compare the pumping power for a rough surface with that for a smooth. To do this, assume that the mass flow necessary if a smooth surface is used corresponds to a Reynolds number \( \text{Re}_s \).

Using the definition of the Reynolds number, eq. (77) with the first term neglected as earlier gives

\[ \frac{\dot{q} \beta_1}{T_w - T_i} = C_{St} \cdot \text{Re}_s^{1+n} \cdot (\text{Re}/\dot{m})^{-1} \]  

(81)

According to the initial assumptions, this quantity, as well as \( \text{Re}/\dot{m} \), is independent of the surface conditions.
From eq. (79) the ratio of the pumping powers for rough and smooth surfaces is then immediately obtained

\[
\frac{P}{P_s} = Re_s \cdot \frac{C_f}{C_{f_s}} \cdot \left( \frac{C_{St}}{C_{St_s}} \right)^\gamma
\]  

(82)

where

\[
\beta = (1+n_s) \left( \frac{3+m_s}{1+n} - \frac{3+m_s}{1+n_s} \right)
\]  

(83)

\[
\gamma = \frac{3+m}{1+n}
\]  

(84)

12.3.2 Experimental results

Assuming that in the case of a smooth surface a Reynolds number of $10^6$ would be necessary (to ensure that the comparison is made in the same range of Reynolds numbers as the measurements), the pumping power ratio has been calculated by eq. (82) for each tested surface using the experimental data for the entire annulus [75].

The results are summarized in table 12.3.1. As can be seen, great savings are possible with rough surfaces, the pumping power being reduced by a factor up to 10.

Table 12.3.1

Comparison of necessary pumping powers with rough or smooth inner surface in an annular heat exchanger

<table>
<thead>
<tr>
<th>Test rod</th>
<th>P/P_s</th>
<th>Test rod</th>
<th>P/P_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.11</td>
<td>R</td>
<td>0.21</td>
</tr>
<tr>
<td>C</td>
<td>0.12</td>
<td>S</td>
<td>0.22</td>
</tr>
<tr>
<td>D</td>
<td>0.12</td>
<td>T_1</td>
<td>0.31</td>
</tr>
<tr>
<td>E</td>
<td>0.17</td>
<td>T_2</td>
<td>0.19</td>
</tr>
<tr>
<td>F</td>
<td>0.14</td>
<td>U</td>
<td>0.15</td>
</tr>
<tr>
<td>G</td>
<td>0.17</td>
<td>V</td>
<td>0.15</td>
</tr>
<tr>
<td>H</td>
<td>0.13</td>
<td>W</td>
<td>0.10</td>
</tr>
<tr>
<td>I</td>
<td>0.17</td>
<td>X</td>
<td>0.10</td>
</tr>
<tr>
<td>K</td>
<td>0.15</td>
<td>Y</td>
<td>0.12</td>
</tr>
<tr>
<td>N</td>
<td>0.17</td>
<td>Å</td>
<td>0.24</td>
</tr>
<tr>
<td>P</td>
<td>0.25</td>
<td>Å</td>
<td>0.17</td>
</tr>
<tr>
<td>Q</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12.4 Comparison of different surface roughnesses for reactor fuel elements, where the geometry can be optimized

12.4.1 Theoretical considerations

Consider a reactor power plant where heat is supplied to the circulating coolant in the fuel elements and energy dissipated to some receiver outside the reactor, either a steam generator or a turbine.

As shown by Margen [38], optimization of a reactor power plant is most conveniently made in several steps. It is assumed that the heat release of the fuel, the coolant flow rate, the inlet temperature to the fuel elements and the core height have been established by a preoptimization. It is further assumed that the design limit for the fuel elements is the surface temperature, which must not exceed a given value.

The required heat transfer coefficient will be given by

\[ \alpha = \frac{q}{UL} \]  \hspace{1cm} (85)

where \( \overline{F} \) is fixed according to the assumptions above.

By definition,

\[ \alpha = \frac{\dot{m}c_p}{A} \cdot St \]  \hspace{1cm} (86)

The pumping power will be obtained as

\[ P = f \cdot \frac{UL}{8A^\frac{3}{2}} \cdot \frac{\dot{m}^3}{\rho^2} \]  \hspace{1cm} (87)

cf. eqs. (73-75).

In limited ranges the friction factor and the Stanton number can be obtained from:

\[ f = C_{fr} \cdot \left( \frac{4\dot{m}}{\pi U} \right)^m \]  \hspace{1cm} (88)
Elimination of the flow area in eq. (87) by eqs. (86) and (85) gives a relation for the pumping power which can be used in eq. (69), and the total cost is found to be

\[ E_{\text{tot}} = \beta_3 \cdot \frac{f}{\text{St}^3} \cdot \frac{1}{U^2L^2} + k_s UL \]  

where

\[ \beta_3 = \frac{k_p \cdot \frac{1}{\rho} \cdot \frac{1}{c_p}}{8 \rho \cdot \frac{1}{c_p} \cdot \frac{1}{\rho}} \]  

The minimum of \( C_{\text{tot}} \) is required under the condition that

\[ \frac{f}{\text{St}^3} \cdot \frac{C_{\text{fr}}}{C_{\text{St}}} \cdot \left( \frac{4 \hat{m}}{11} \right)^{m-3n} U^{3n-m} = 0 \]  

which is obtained by combination of eqs. (88) and (89).

Using the method for determination of maxima and minima under side conditions it is found that the lowest cost is obtained for

\[ U = \frac{1}{L} \cdot \frac{1}{k_s} \cdot \beta_3 \cdot \frac{1}{\text{St}^3} \cdot f^{1/3} \]  

and the minimum cost is found to be

\[ E_{\text{tot}} = (2-3n+m)^{-2/3} \cdot (\beta_3 \cdot k_s^2)^{1/3} \cdot \frac{f^{1/3}}{\text{St}^3} \]  

where \( f/\text{St}^3 \) is to be determined from eqs. (92) and (93). By combination of these equations it is found that \( f/\text{St}^3 \) is to be calculated for a Reynolds number

\[ \text{Re} = \left[ \frac{4 \hat{m}}{11} \cdot L \cdot \left( \frac{k_s}{\beta_3} \right)^{1/3} \cdot \frac{C_{\text{St}}}{C_{\text{fr}}} \cdot (2-3n+m)^{-1/3} \right]^{1-n+\frac{m}{3}} \]  

The ratio of the costs for a smooth or rough surface is obtained as
\[
\frac{E_s}{E_r} = \left( \frac{2-3n + m_r}{2-3n + m_s} \right)^{2/3} \cdot \frac{St_r}{St_s} \cdot \left( \frac{f_{r}}{f_{s}} \right)^{1/3}
\]  
\text{(96)}

where the Stanton number and friction factor are to be calculated at different Reynolds numbers for the smooth and the rough surface according to eq. (95).

If the optimum Reynolds number, \(Re_s\), for a smooth surface is known, eq. (95) can be utilized for calculation of the group \(\frac{4\pi n}{\eta L} \left( \frac{k_s}{\rho_s} \right)^{1/3}\). The optimal Reynolds number \(Re_r\) for a rough surface is then obtained as

\[
Re_r = Re_s \left[ 1 - n + \frac{m_s}{3} \right]^{-1-n} \left[ \frac{C_{fr_r}}{C_{fr_s}} \left( \frac{C_{fr_s}}{C_{fr_r}} \left( \frac{2-3n + m_s}{2-3n + m_r} \right)^{1/3} \right) \right]^{1-n} \left[ \frac{m_r}{3} \right]
\]  
\text{(97)}

12.4.2 Experimental results

By calculation of friction factors and Stanton numbers for the different types of roughness tested starting from the friction and heat transfer similarity laws at different Reynolds numbers, it was possible to find values for the constants \(C_{fr}\) and \(C_{St}\) and the powers \(m\) and \(n\) in the approximate relations (88) and (89). Assuming then for a smooth surface \(C_{fr} = 0.184\), \(C_{St} = 0.029\), \(m=n=-0.2\) (valid for a smooth circular channel rather than for the inner channel of an annulus, cf. fig. 15, which shows higher friction) and postulating that the optimum smooth Reynolds number is \(2.8 \cdot 10^5\) (giving the optimal Reynolds number for a rough surface as about \(4 \cdot 10^5\)) it was possible to calculate the cost ratio from eq. (96).

The results are given in table 12.4.1. All the tested roughnesses are superior to a smooth surface. Gains of up to 40\% are possible.

This demonstrates clearly that improved performance of reactor fuel elements can be obtained by use of rough surfaces.
### Table 12.4.1

The ratio of the costs for smooth and different kinds of rough surfaces, $E_s/E_r$

#### Rectangular fins, $s/b = 8$

<table>
<thead>
<tr>
<th>$h/D_h$</th>
<th>(s-b)/h</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003</td>
<td>1.26</td>
<td>1.25</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>1.29</td>
<td>1.28</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>0.010</td>
<td>1.32</td>
<td>1.30</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>0.015</td>
<td>1.33</td>
<td>1.32</td>
<td>1.31</td>
<td></td>
</tr>
</tbody>
</table>

#### Various shapes, $h/D_h = 0.01$

<table>
<thead>
<tr>
<th>Shape</th>
<th>$b/h$</th>
<th>$s-b/h$</th>
<th>$E_s/E_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>11.6</td>
<td>11.8</td>
<td>1.15</td>
</tr>
<tr>
<td>&quot;</td>
<td>3.9</td>
<td>13.7</td>
<td>1.43</td>
</tr>
<tr>
<td>&quot;</td>
<td>2.9</td>
<td>5.0</td>
<td>1.24</td>
</tr>
<tr>
<td>&quot;</td>
<td>1.0</td>
<td>26.8</td>
<td>1.10</td>
</tr>
<tr>
<td>Sawtooth, cutting edge upstream</td>
<td>-</td>
<td>8.5</td>
<td>1.25</td>
</tr>
<tr>
<td>&quot; &quot;</td>
<td>-</td>
<td>15.0</td>
<td>1.17</td>
</tr>
<tr>
<td>Sawtooth, cutting edge downstream</td>
<td>-</td>
<td>15.0</td>
<td>1.06</td>
</tr>
<tr>
<td>Sinus</td>
<td>-</td>
<td>7.9</td>
<td>1.14</td>
</tr>
<tr>
<td>Rounded bottom</td>
<td>-</td>
<td>14.1</td>
<td>1.06</td>
</tr>
</tbody>
</table>

### 13. APPLICATION OF THE DATA TO FUEL ELEMENT CALCULATIONS

Calculation of the friction factor and Stanton number for a subchannel in a rod cluster from the data in this work requires that an equivalent annular channel is first found. It is suggested that the zero shear radius of this annulus is calculated from the zero shear line in the rod bundle by the equivalent diameter concept, and that the outer radius in the annulus is calculated by the same method applied to
the outer subchannel of the annulus and the channel between the zero shear line and the surrounding rods. Calculations show that the choice of the outer radius of the equivalent annulus is not critical.

The constants $f(h^+)$ and $G'_D$ must then be determined for the particular roughness geometry. In the case of rectangular fins figs. 21 and 22 can be used. For other shapes systematic information is not available and data are only known for the geometries tested. For these, the information can be obtained from table 10.2.1.

Knowing the fin height, the friction factor can then be determined immediately from eq. (26), where $(K_f)^h_1$ is to be calculated from eq. (20) with the integration performed from $r_1^+h$ to $r$.

The function $g(h^+, Pr)$ was determined in the experiments for $Pr = 0.7$, and the constant $G'_D$ includes the effect of the Prandtl number.

Assuming that the Prandtl number dependence is the same as that determined by Dipprey and Sabersky [10], rearrangement of eq. (48) and insertion of eq. (50) gives the Stanton number as

$$St_1 = \frac{f_1}{\delta}$$

$$= \frac{f_1}{1 + \sqrt{\frac{f_1}{8} \left[ G'_D \cdot \left( \frac{h}{D_h} \cdot \text{Re} \sqrt{\frac{f_1}{8}} \right)^{0.2} \left( \frac{Pr}{0.7} \right)^{0.44} \right]}}$$

(98)

where $f_1$ of course is the friction factor determined previously.

The advantages of the method are obvious. $f(h^+)$ and $G'_D$ will be the same for the whole cluster. If a computer is used for the calculations, they can be given as input data and the functions (26) and (98) can be included in the program.

If data is not available for the particular roughness geometry of interest, $f(h^+)$ and $G'_D$ can be determined by in principle only one experiment. The variation of the friction factor and the Stanton number with the ratio $h/D_h$ and the Reynolds number is then described by the friction and heat transfer similarity laws, which eliminates the fitting of polynomial or exponential functions to experimental data. That was earlier necessary.
14. CONCLUSIONS

1. Where comparisons can be made with earlier investigations, agreement is satisfactory.

2. The relation of Nunner [45] may be sufficiently accurate for many applications.

3. The friction and heat transfer similarity laws offer a comparatively accurate method for correlation of rough surface data, and make considerable reductions of future experiments possible.

4. In heat exchangers with a limited surface temperature considerable gains in economy can be obtained by the use of rough heat transfer surfaces.

15. ACKNOWLEDGEMENTS

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16. NOMENCLATURE

A  flow area
B  geometrical parameter eq. (22)
b  fin width
C  constant
c_p specific heat
D_h equivalent diameter
d  diameter
C  cost
F  function
f  friction factor
f(h^+) function eq. (24)
G'_FR constant eq. (50)
G''_FR constant, G''_FR = G'_FR · Pr^q
q(h^+ , Pr) function eq. (29)
h  fin height
K  geometrical function eq. (20)
k  constants in the
k_p  cost function eq. (69)
L  length
m  exponent
m  mass flow
n  exponent
P  pumping power
Pr Prandtl number
p  pressure (or exponent)
q  exponent
q  heating rate
q'' surface heat flux
Re Reynolds number
r  radius
r_b radius at which the velocity equals the bulk value
v  recovery factor
St Stanton number
s  fin pitch
T  absolute temperature
t  temperature
U \text{ perimeter}

\frac{\partial u}{\partial x} \text{ axial velocity}

x \text{ axial coordinate}

\alpha \text{ heat transfer coefficient}

\beta \text{ exponent eq. (83)}

\beta_1, \beta_2, \beta_3 \text{ constants eqs. (78), (80) and (91)}

\gamma \text{ exponent eq. (84)}

\delta \text{ thickness of laminar sublayer}

\delta_T \text{ distance from the wall at which viscous effects become negligible, i.e. where the turbulent core begins}

\epsilon \text{ emissivity}

\epsilon_H \text{ eddy diffusivity of heat}

\epsilon_M \text{ eddy diffusivity of momentum}

\Delta \text{ difference}

\eta \text{ dynamic viscosity}

\phi \text{ temperature difference}

k, k_0 \text{ constants}

\Lambda \text{ equivalent turbulent thermal conductivity}

\lambda \text{ thermal conductivity}

\rho \text{ density}

\sigma \text{ standard deviation}

\Psi \text{ shape factor}

\textbf{Subscripts}

0 \text{ surface of zero shear}

1 \text{ inner surface of annulus or inner subchannel}

2 \text{ outer surface of annulus or outer subchannel}

(1.0) \text{ at } b/h = 1.0

1 \text{ measuring station I 2370 mm from the inlet}

II \text{ " " II 2870 " " " "}

III \text{ " " III 3370 " " " "}

aw \text{ adiabatic wall}

b \text{ bulk value}

calc \} \text{ calculated}
co convection
exp experimental
G Gargaud [16]
fr friction
i inlet
msd measured
s smooth surface
St Stanton number
T at $\delta_T$
t stagnation
tot total
u outlet
W Wilkie [68, 69]
w wall

**Special signs**

— mean value, e.g. $\bar{u}$
∧ at radius of maximum velocity
∨ minimum value
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APPENDIX

Eccentricity of the test rod

The observed mean eccentricities and the variance of the eccentricity for the different test rods are given in the table below.

The eccentricity has been defined as

\[ E = \left| \frac{2y - (d_2 - d_1)}{d_2 - d_1} \right| \]

where \( y \) is the distance from the shroud to the test rod.

<table>
<thead>
<tr>
<th>Test rod</th>
<th>( \bar{E} )</th>
<th>( \sigma_E )</th>
<th>Test rod</th>
<th>( \bar{E} )</th>
<th>( \sigma_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.046</td>
<td>0.022</td>
<td>P</td>
<td>0.048</td>
<td>0.009</td>
</tr>
<tr>
<td>B</td>
<td>0.034</td>
<td>0.002</td>
<td>Q</td>
<td>0.048</td>
<td>0.005</td>
</tr>
<tr>
<td>C</td>
<td>0.059</td>
<td>0.014</td>
<td>R</td>
<td>0.034</td>
<td>0.007</td>
</tr>
<tr>
<td>D</td>
<td>0.035</td>
<td>0.012</td>
<td>S</td>
<td>0.060</td>
<td>0.005</td>
</tr>
<tr>
<td>E</td>
<td>0.037</td>
<td>0.01</td>
<td>T_1</td>
<td>0.049</td>
<td>0.005</td>
</tr>
<tr>
<td>F</td>
<td>0.048</td>
<td>0.009</td>
<td>T_2</td>
<td>0.056</td>
<td>0.004</td>
</tr>
<tr>
<td>G</td>
<td>0.046</td>
<td>0.004</td>
<td>U</td>
<td>0.043</td>
<td>0.008</td>
</tr>
<tr>
<td>H_2</td>
<td>0.047</td>
<td>0.002</td>
<td>V</td>
<td>0.041</td>
<td>0.002</td>
</tr>
<tr>
<td>I</td>
<td>0.067</td>
<td>0.010</td>
<td>W</td>
<td>0.034</td>
<td>0.004</td>
</tr>
<tr>
<td>J</td>
<td>0.050</td>
<td>0.001</td>
<td>X</td>
<td>0.030</td>
<td>0.010</td>
</tr>
<tr>
<td>K</td>
<td>0.041</td>
<td>0.008</td>
<td>Y</td>
<td>0.053</td>
<td>0.004</td>
</tr>
<tr>
<td>N</td>
<td>0.047</td>
<td>0.003</td>
<td>( \ddot{\text{A}} )</td>
<td>0.022</td>
<td>0.009</td>
</tr>
</tbody>
</table>

\( \bar{E} \) is the mean value and \( \sigma_E \) the standard deviation of the eccentricity for each test rod.

As can be seen, the eccentricity is generally of the order of 5\% or less and was in all cases less than 8\%. This is tolerable according to Burgoyne et al. [7], a conclusion which is also supported by measurements of Nöthiger [46] and Judd and Wade [26] on heat transfer coefficients in eccentric annuli.
Explanations to fig. 1

1. Filter
2. Inlet box
3. Blower
4. Inlet throttle valve
5. Guide vanes
6. Flow rectifier
7. Measuring probes
8. Foster flow meter
9. Expansion bellow
10. Electric connection
11. Expansion bellow
12. Electric connection
13. Mixing chamber
14. Measuring probes
15. Throttle valve
16. Outlet box
17. Combination probe, traversable
18. Fixed stagnation pressure probe (only at x = 3390)
19. Stagnation pressure probe, traversable (only at x = 2890)
20. Temperature probe, fixed
21. Electric insulation
22. Test rod
23. Shroud tube
24. Distance holder
25. Ceramic tube
26. Thermocouple
27. Brass tube
28. Air gap 0.5 mm
29. Copper ring
The atmospheric heat transfer rig

FRIGGA III
Fig. 2 Roughened test rod.
Thermocouple Cr-Al

Fig. 3 Combination probe for velocity and temperature traverses.
Fig. 4 Tested roughness geometries.
Test rod: D

Test rod: E

Test rod: F

Test rod: G

Test rod: H

Fig. 5 Tested roughness geometrics.
Test rod: I

Test rod: J

Test rod: P

Test rod: Q

Test rod: U

Fig. 6 Tested roughness geometries.
Test rod: N

Test rod: R

Test rod: S

Test rod: T

Test rod: V

Fig. 7 Tested roughness geometries.
Fig. 8 Tested roughness geometries.
Fig. 9 Friction factor measured in the smooth circular channel obtained when the test rod is removed.
Symbols

- $10^{-5}$ Re
  - $1.6-1.8$
  - $2.9-3.2$
  - $4.3-5.3$

**Test rod: A**

(smooth)

**Test rod: B**

$10^3$ St

$x/D_h$

$10^3$ St

$x/D_h$

Test rod: K

$10^3$ St

$x/D_h$

Test rod: N

$10^3$ St

$x/D_h$

Test rod: X

$10^3$ St

$x/D_h$

Test rod: A

$10^3$ St

$x/D_h$

Fig. 10

Observed axial variation of the Stanton number.

- Test rod: G
- Test rod: V
- Test rod: K
- Test rod: X
- Test rod: N
Adiabatic flow
Heat transfer from inner surface

Prediction of Gunn and Darling [20] for annular channel

Relation for circular channels, eq. (63)

10^2 f

10^{-5} \text{Re}

1
2
3
4
5

10^3 St

10^{-5} \text{Re}

1
2
3
4
5

Fig. 11 Total passage friction factor and Stanton number for the smooth annulus.
Fig. 12 Comparison of experimental data for the radius of maximum velocity by turbulent flow in annuli.
Fig. 13 Velocity and temperature distributions measured in smooth rod experiments.
Fig. 14 Velocity and temperature distributions measured in rough rod experiments.
Fig. 15. Diameter of maximum velocity, transformed friction factor and Stanton number vs. transformed Reynolds number for the smooth annulus.
f(h^+) g(h^+, Pr)

Test rod f(h^+) g(h^+, Pr)
X •
Y •

a. Fins with (s-b)/h = 1.6; b/h = 0.25

f(h^+) g(h^+, Pr)

Test rod f(h^+) g(h^+, Pr)
C •
W •

b. Fins with (s-b)/h = 3.4; b/h = 0.55

Fig. 16. f(h^+) and g(h^+, Pr) vs. h^+ for rectangular fins with s/b = 8.
Fig. 17. $f(h^+)$ and $g(h^+, Pr)$ vs. $h^+$ for rectangular fins with $s/b = 8$.
a. Fins with s/b = 8. Other dimensions are given in table 4.3.1

f(h⁺)  g(h⁺, Pr)

b. Fins with various s/b. All dimensions are given in table 4.3.1

Fig. 18. f(h⁺) and g(h⁺, Pr) vs. h⁺ for rectangular fins.
Fig. 19. $f(h^+)$ and $g(h^+, Pr)$ vs. $h^+$ for different finshapes.

- **Part a.** Fins of sawtooth shape
- **Part b.** Fins with a rounded bottom (P) and of sinus shape (G)
Fig. 20. \( f(h^+) \) and \( G'_{FR} \) vs. \( (s-b)/h \) for rectangular fins with \( s/b = 8 \).
The circles show the dimensions of the fins tested. The number beside each circle refers to the observed value of \( f(h^+) \). Black circles refer to fins with \( s/b = 8 \) for which data are given in fig. 20.

The curves for constant \( f(h^+) \) must be considered as approximate.

Fig. 21. \( f(h^+) \) for rectangular fins.
The circles show the dimensions of the fins tested. The number beside each circle refers to the observed value of $G'_F$. Black circles refer to fins with $s/b = 8$ for which data are given in fig. 20.

The curves for constant $G'_F$ must be considered as approximate.

Fig. 22. $G'_F$ for rectangular fins.
Fig. 23. Comparison of experimental and calculated friction factors for rectangular fins with $a/b = 8$. 
Fig. 24. Comparison of experimental and calculated Stanton numbers for rectangular fins with $s/b = 8$. 

$10^3 St_{\text{calc}}$

$10^3 St_{\text{exp}}$
Fig. 25. Calculated relative friction factors and Stanton numbers for rectangular fins with $s/b = 8$. 
Fig. 26 Comparison of measured and calculated velocities in the inner subchannel of smooth annuli.
Fig. 27 Comparison of measured and calculated velocities in the inner subchannel of partially rough annuli.
Fig. 28. Comparison with data of Wilkie [68] for square fins with \((s-b)/h = 6.2\).
This investigation:

- Indicates b/h and s/h where experiments were made. (At a different h/D_h.)
- Calculated curve. The influence of h/D_h has been estimated with the friction similarity law.

Fig. 29. Influence of the fin width on the friction factor for rectangular fins with h/D_h = 0.01026.
Fig. 30. Comparison of experimental relative Stanton numbers with values calculated from the theory of Nunner [45], eq. (68).