Spatially and Temperature Dependent Dancoff Method for LWR Lattice Physics Code

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Spatially and temperature dependent self-shielding method SDDM was developed, and validated by comparing its reactivity changes due to fuel temperature with those obtained from the continuous Monte Carlo code calculation.

1. Introduction

To evaluate the temperature profile and fuel integrity parameters, the fuel performance codes need an accurate radial power distribution within the fuel rod. The spatial variation of isotopic contents in a fuel rod has to be taken into account in order to compute reliable radial power distributions. For that purpose, the micro-depletion, spatial dependent resonance self-shielding and multi-ring flux solver became the necessary modules in any modern lattice code.

In 1996, C. C. Stoker and Z. J. Weiss proposed a method by which shielded resonance cross sections can be generated for any number of concentric rings in a fuel pin using pre-tabulated resonance integrals [1]. Combining their idea and the conventional method widely employed in LWR lattice physics codes such as PHOENIX-P [2], the spatially dependent Dancoff method (SDDM) was newly developed as the generalization of the conventional Dancoff method. SDDM method is described in Ref. [3].

In this paper we will extend SDDM to take into account the temperature profile in the fuel rod. In the conventional lattice physics codes, a fuel lump has usually been treated as one resonant region with a flat temperature profile. This flat temperature is usually referenced as the effective temperature that can reproduce the reaction rates of multi-ring with temperature profile.

With the new enhancement of SDDM we will be able to predict the spatial power within a fuel rod considering the variation of isotopic contents, the spatial dependent self-shielding and the temperature profile within the rod.

2. Spatially Dependent Resonance Shielding Method with Temperature Profile

As described in Ref. [3], SDDM was developed so as to predict the power profile within fuel rods in a LWR fuel assembly. It was the generalization of the conventional Dancoff method. In the case where the flat temperature called the effective temperature \( T_{\text{eff}} \) is...
assumed, the spatially dependent Dancoff method (SDDM) has been validated by comparing the spatial power, variations of reaction rates and burnup distribution obtained from SDDM with a Monte Carlo result and destructive measurements. From the viewpoint of micro-nuclear physics, the temperature profile has to be considered in the evaluation. In this section, temperature dependent SDDM for such evaluation is to be derived.

In SDDM, the blackness of a concentric fuel ring is generated using Eq. (1) below,

$$ \gamma_i = \gamma(\rho_i) - \gamma(\rho_{i-1}) = \left( \gamma_i^{BC} - \gamma_i^{AB} \right) - \left( \gamma_i^{BC} - \gamma_i^{AB} \right) $$

(1)

The meanings of the notations are shown in Fig. 1. When a temperature profile is assumed, Eq. (2) can represent the appropriate blackness of the ring $i$,

$$ \gamma_i = \left\{ \gamma_i^{BC}(T_i) - \gamma_i^{AB}(T_i) \right\} - \left\{ \gamma_i^{BC}(T_{i-1}) - \gamma_i^{AB}(T_{i-1}) \right\} $$

(2)

The $\gamma_i^{BC}$ and $\gamma_i^{AB}$ are temperature dependent and have their own temperature profile within the region, but the regions except for the target ring region $i$ can be canceled out using Eq. (2) above.

Therefore, using SDDM procedure described in Ref.[3], the escape probability of fuel ring I within a fuel rod in a fuel assembly can be obtained from,

$$ P_{esc} = \sum_{m=1}^{4} F_m \sum_{n=1}^{2} |\ell_m| \sum_{j} (T_j) + \alpha_n $$

(3)

where,

$$ \alpha_{1,2} = \frac{(5C + 6) \pm \sqrt{C^2 + 36C + 36}}{2(C + 1)} $$

(4)

$$ \beta_1 = \frac{4C + 61}{C + 1} - \alpha_1 $$

(5)

$$ \beta_2 = 1 - \beta_1 $$

(6)

$F_m$ is the same parameter as described in Ref.[1,3]. $\ell_i$ is a mean chord length of OAB or OBC as shown in Fig. 1. It can be generated using Eq.(7).

$$ \ell_i^{BC} = \frac{2R}{\pi} \left( \sqrt{1 - \rho_i^2} + \frac{1}{\rho_i} \sin^{-1} \rho_i \pm \frac{\pi}{2} \rho_i \right) $$

(7)

Once the escape probability is obtained from Eq. (3), the neutron spectrum can be written as,

$$ \phi_i(E) = \frac{1}{E} \sum_{m=1}^{4} F_m \sum_{n=1}^{2} |\ell_m| \sum_{j} (T_j) + \alpha_n $$

(8)

by using the slowing down equation.

Therefore, we can get the effective resonance shielding cross section as

$$ \sigma_{x,i}^{\nu}(T_i) = \frac{\sum_{m=1}^{4} F_m \sum_{n=1}^{2} \beta_n I_{x,g}^{k} (\sigma_{b,j}^{\nu m}, T_i)}{1 - \sum_{m=1}^{4} F_m \sum_{n=1}^{2} \beta_n I_{p,g}^{k} (\sigma_{b,j}^{\nu m}, T_i) \sigma_{b,i}^{\nu m}} $$

(9)

where,
\[
\sigma_{b}^{\text{unk}} = \frac{\sum_{b} + \alpha_{b} I_{\epsilon_{m}}}{N_{k}}
\]  

(10).

Therefore, the spatial and temperature dependent effective resonance cross-section can be easily obtained with using pre-tabulated resonance integrals \( I(\sigma_{b},T) \).

In practical core design calculation, fuel pellet is treated as one resonant region. In such case, effective temperature is used so as to preserve total reaction rates of detail mode such as multiple fuel ring model. To obtain such effective temperature, several methods, for example “volume average,” and “chord average,”[4] were developed. Now, by using SDDM, the effective temperature can be determined.

In the case of one resonant fuel rod, SDDM equation corresponds completely to the conventional resonance cross-section if a flat temperature is assumed,

\[
\sigma_{x}^{T}(T) = \frac{\sum_{n=1}^{2} \beta_{n} I_{a,\epsilon_{g}}^{k}(\sigma_{b}^{\text{unk}},T)}{1 - \sum_{n=1}^{2} \beta_{n} I_{a,\epsilon_{g}}^{k}(\sigma_{b}^{\text{unk}},T)}
\]

(11).

Note that the quantity to be preserved is the total reaction rate within the fuel rod. Therefore, applying one-term relation, the effective temperature has to satisfy,

\[
I_{x}^{k}(\sigma_{b}^{\text{eff}},T_{\text{eff}}) = \frac{\sum_{i} V_{i} \sum_{m=1}^{2} F_{m} \sum_{n=1}^{2} \beta_{n} I_{a,\epsilon_{g}}^{k}(\sigma_{b}^{\text{unk}},T)}{\sum_{i} V_{i}}
\]

(12).

### 3. Numerical Results

Comparison of a typical pin cell calculation between a flat temperature model and a temperature profile model was performed. A continuous energy Monte Carlo code MVP developed by JAERI was used to get reference solutions to be compared with SDDM. The SDDM has been implemented in PARAGON[5,3] code, jointly developed by Westinghouse and MHI. The pin cell stands for a typical PWR 17x17 assembly and its specifications are shown in Table 1.

<table>
<thead>
<tr>
<th>Cell Pitch</th>
<th>1.33(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pellet Outer Radius</td>
<td>0.41(cm)</td>
</tr>
<tr>
<td>Cladding Outer Radius</td>
<td>0.48(cm)</td>
</tr>
<tr>
<td>U-235 Enrichment</td>
<td>4.1(%)</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Table 2 Doppler Defect from Zero Power (cold) to Full power conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Doppler Defect (pcm)</strong></td>
</tr>
<tr>
<td>PARAGON(SDDM)</td>
</tr>
<tr>
<td>MVP</td>
</tr>
<tr>
<td>Difference (%)</td>
</tr>
</tbody>
</table>

Table 2 shows the reactivity differences between 0% power and 100% power cases. The temperatures of the 0% power (cold) are all 300(K) for fuel pellet, cladding and moderator regions. Physical temperature used in the 100% power case came from a fuel performance code. The cross-section library used in the comparison is ENDF/B6[7,8]. Change of moderator density due to the power was not taken into account so as to make the effect of the temperature profile clear. The Doppler defects from the zero power to the full power are 2160 and 2215 pcm for MVP and PARAGON(SDDM), respectively. Therefore, it can be seen that the reactivity change (Doppler) of SDDM agrees well with the MVP result.
Figure 2 and Figure 3 show reaction rate distributions of U-238 absorption within the fuel rod. Figure 2 shows the zero power result and Figure 3 shows the full power result. The distributions of PARAGON with SDDM agree well to those of MVP within about 3% in both cases. The tendency of the difference between MVP and PARAGON is almost identical for both Zero and Full power cases. From the comparisons of reactivity and reaction rate distribution, it is found that the effect of the temperature profile on U-238 reaction rate distribution can be captured by using SDDM.

As described in the last paragraph of the previous section, one resonant fuel rod model is appropriate for core design calculation from the viewpoint of computing time. The reaction rate obtained from the multi-ring model should be preserved in the one resonant region case. To do so, the effective temperature has been used in the design calculation, and enough accuracy of Doppler defect and coefficient has been confirmed by many startup tests of PWRs. However, it can be expected to make the effective temperature more accurate by using Eq.(12).

To validate the Eq.(12), three pin cell calculations were compared. One is the pin cell model with 10 rings shown in the previous Figure 3. The second one is the pin cell model with the volume average temperature of the Full power. The last one is the pin cell model with the effective temperature obtained from Eq.(12). Table 3 shows the reactivity difference from the reference case (multi-ring with the temperature profile). It is found that the effective temperature obtained from Eq.(12) gives almost same reactivity as the detailed one (the first case).

On the other hand, using the continuous Monte Carlo code MVP, the effective temperature becomes the volume average one. It was also reported in Ref.[9]. The difference between the
volume average temperature and the effective temperature in SDDM cases came from the difference of the denominators of Eq.(9) and Eq.(11). We need to investigate the reason why the effective temperature obtained with the continuous Monte Carlo code corresponds to the volume average temperature.

4. Conclusion

Spatially and temperature dependent self-shielding method SDDM was developed. The SDDM was validated by comparing its reactivity changes due to fuel temperature with that obtained from the continuous Monte Carlo code MVP calculation. The method to generate the effective flat temperature that can preserve total reaction rate within a rod was also developed by using SDDM. Although one needs to do more study as a function of burnup and other parameters (enrichment, rod pitch, fuel type etc) before concluding on this issue, we can get good agreement of temperature dependent reactivity between one region model and multi-ring model by using the effective flat temperature obtained with SDDM.

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References