SUBCRITICALITY CALCULATION IN NUCLEAR REACTORS WITH EXTERNAL NEUTRON SOURCES

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ABSTRACT

The main objective of this paper consists on the development of a methodology to monitor subcriticality. We used the inverse point kinetic equation with 6 precursor groups and external neutron sources for the calculation of reactivity. The input data for the inverse point kinetic equation was adjusted, in order to use the neutron counting rates obtained from the subcritical multiplication (1/M) in a nuclear reactor. In this paper, we assumed that the external neutron sources strength is constant and we define it in terms of a known initial condition. The results obtained from inverse point kinetic equation with external neutron sources were compared with the results obtained with a benchmark calculation, and showed good accuracy.

1. INTRODUCTION

In the operation of a nuclear plant, especially after core reload, it is necessary to monitor of the neutron multiplication in the reactor core. The monitoring and the prediction of the criticality condition can be made as a function of the control rod banks withdrawal for insertion of reactivity. The prediction of the neutron multiplication factor can be carried out through the subcritical multiplication [1], as a function of the control rod positions during the start up of the reactor.

With relation to the safety of the reactor core, subcriticality monitoring should be made correctly to prevent certain unexpected accidents, mainly of reactivity insertion. In this way, we will use the inverse point kinetic equation [2] with external neutron sources to follow up the neutron multiplication in the reactor through reactivity calculation. The input data for the inverse point kinetic equation was adjusted, in order to use the neutron counting rates.

In this paper, we implement the finite difference method to discretize the derivative of the neutron density in the reactor and the Runge-Kutta method to approximate the integral of the power history using the Leibniz integral rule [3].
2. THE POINT KINETIC EQUATIONS WITH EXTERNAL NEUTRON SOURCES

The point kinetic equations constitute a set of coupled ordinary differential equations, where the only time-dependent kinetic parameter is the reactivity $\rho(t)$. By this set of equations, one can investigate the behavior of the neutron population, due to a small perturbation caused in the reactor core by an additional reactivity insertion that can be related to the movement of the control rod banks during the operation of the nuclear reactor or to the variation of boron concentration in the primary circuit, among others.

We assume that the external neutron source strength is constant since the start up of the reactor, in particular, after the core reload until criticality condition is achieved. With the external neutron sources, the point kinetic equations assume the following form:

$$\frac{d}{dt} n(t) = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i c_i(t) + S$$,

(1)

$$\frac{d}{dt} c_i(t) = \frac{\beta_i}{\Lambda} n(t) - \lambda_i c_i(t), \quad i = 1,...,6$$.

(2)

The initial precursor concentrations $c_{i,o}$ in the reactor, can be obtained from a known initial steady state condition, obtained from equation (2),

$$c_{i,o} = \frac{\beta_i}{\lambda_i \Lambda} n_o$$,

(3)

where $n_o$ is neutron density at the initial state, $\lambda_i$ is the $i$th precursor group decay constant, $\Lambda$ is the generation time and $\beta_i$ is the $i$th group delayed neutron fraction.

The effective strength of the external neutron sources can be obtained from the initial steady state condition of equation (1),

$$S = -\frac{\rho_o}{\Lambda} n_o$$,

(4)

where $\rho_o$ is the initial reactivity of the system. The external neutron sources in the core produce neutrons in all directions with a rate of $S_o$ neutrons/s and they are located in specific regions of the reactor.

The system of equations (1) and (2) can be reduced to an integro-differential equation for $n(t)$. For this, it is enough to apply the inverse method on the point kinetic equations [2]. Firstly, we solve the set of ordinary differential equations for the precursor concentrations. Multiplying the equation (2) by the integrating factor $e^{\lambda t}$, integrating the resulting equation
and substituting the initial condition given by the equation (3), the precursor concentrations can be written as:

$$c_i(t) = \frac{\beta_i}{\Lambda} \left( n_o - \frac{1}{\lambda_i} \int_0^t \left( n(t') e^{-\lambda_i(t-t')} \right) dt' \right) .$$  \hfill (5)

Substituting equations (4) and (5) in the equation (1), we obtain,

$$\frac{d}{dt} n(t) = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^6 \beta_i \left( n_o - \frac{1}{\lambda_i} \int_0^t \left( n(t') e^{-\lambda_i(t-t')} \right) dt' \right) - \frac{\rho_o}{\Lambda} n_o .$$  \hfill (6)

From equation (6) it is possible to obtain an equation that allows us to express the time-dependent reactivity [2] in terms of the neutron density $n(t)$.

$$\rho(t) = \beta + \frac{\Lambda}{n(t)} \frac{d}{dt} n(t) - \sum_{i=1}^6 \beta_i \left( n_o - \frac{1}{\lambda_i} \int_0^t \left( n(t') e^{-\lambda_i(t-t')} \right) dt' \right) + \frac{n_o}{n(t)} \rho_o .$$ \hfill (7)

### 3. ADJUST OF INPUT DATA IN THE INVERSE POINT KINETIC EQUATION

For the monitoring of the neutron subcritical multiplication in the reactor, we must make some adjustments in the neutron counting rates. If the second and third terms of the right-hand side of equation (7) are divided and multiplied by a reference value of the neutron density $n_{ref}$, one has:

$$\rho(t) = \beta + \frac{\Lambda}{n(t)} \frac{d}{dt} \left[ \ln \left( \frac{n(t)}{n_{ref}} \right) \right] - \sum_{i=1}^6 \frac{\beta_i}{n(t)/n_{ref}} \left( n_o - \frac{1}{\lambda_i} \int_0^t \left( \frac{n(t')}{n_{ref}} e^{-\lambda_i(t-t')} \right) dt' \right) + \frac{n_o/n_{ref}}{n(t)/n_{ref}} \rho_o .$$  \hfill (8)

Note that equation (8) allows us to use the definition of the neutron counting rates in terms of the subcritical multiplication factor $M$ [4]. Since the eigenvalue is known, the subcritical multiplication factor can be determined by the following equation:

$$M = \frac{n}{n_{ref}} = \frac{1}{1-k_{eff}} ,$$  \hfill (9)

where the value $k_{eff}$ can be obtained by solving the diffusion equation.

Rewriting the subcritical multiplication $M(t)$ as a function of the time, the reactivity of a subcritical system can be obtained in terms of the following equation:

$$\rho(t) = \beta + \frac{\Lambda}{d/dt} \left[ \ln M(t) \right] - \sum_{i=1}^6 \frac{\beta_i}{M(t)} \left( M_o - \frac{1}{\lambda_i} \int_0^t \left( M(t') e^{-\lambda_i(t-t')} \right) dt' \right) + \frac{M_o}{M(t)} \rho_o .$$  \hfill (10)
where $M_o$ is the ratio of neutron initial counting rates in the reactor.

### 4. NUMERICAL METHOD TO CALCULATE REACTIVITY

The point kinetic equations have been presented as a model to describe the time behavior of the neutron population in a nuclear reactor.

The proposed method to discretize this system of equations is known as finite difference method [5]. This method consists on approximating the differential of a certain function $f(t)$ in the interval $[t, t + \Delta t]$, by:

$$\frac{df}{dt} \approx \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$  \hspace{1cm} (11)

It is important to remember that the larger the number of points (smaller $\Delta t$) in the interval $[t, t + \Delta t]$, the smaller the error will be, causing the numerical solution to be closer to the analytical solution.

Assuming that the neutron density is constant in the interval $-\infty < t' < 0$, the precursor concentrations given by the equation (5), can be rewritten in the following way:

$$c_i(t) = \frac{\beta_i}{\Lambda} \int_{-\infty}^{t'} n(t')e^{-\lambda_i(t-t')} dt'.$$  \hspace{1cm} (12)

To simplify notation, we will use the definition of power history,

$$H_i(t) \equiv \int_{-\infty}^{t'} n(t')e^{-\lambda_i(t-t')} dt'.$$  \hspace{1cm} (13)

We use the 4th-order Runge-Kutta method [6] to numerically approximate the integral of the power history. In order to apply the Runge-Kutta method we need to transform the power history, given by the equation (13), to an ordinary differential equation. To this, we use the Leibniz rule [3],

$$\frac{d}{dt} \int_{a(t)}^{b(t)} f(t') dt' = \int_{a(t)}^{b(t)} \frac{d}{dt} f(t') dt' + f[b(t)] \frac{db(t)}{dt} - f[a(t)] \frac{da(t)}{dt}.$$  \hspace{1cm} (14)

Applying the Leibniz rule on the power history given by the equation (13), we have,

$$\frac{d}{dt} H_i(t) = n(t) - \lambda_i H_i(t), \quad i = 1, \ldots, 6.$$  \hspace{1cm} (15)

The set of equations (15) constitutes a system of ordinary differential equations for the power history. To solve these equations, we will apply the Runge-Kutta method, whose algorithm RK4 is given by:
\[ k_1 = n(t) - \lambda_i H_i(t) \]
\[ k_2 = n(t) - \lambda_i \left[ H_i(t) + \left( \Delta t / 2 \right) \cdot k_1 \right] \]
\[ k_3 = n(t) - \lambda_i \left[ H_i(t) + \left( \Delta t / 2 \right) \cdot k_2 \right] \]
\[ k_4 = n(t) - \lambda_i \left[ H_i(t) + \Delta t \cdot k_3 \right] \]

where the power history, rewritten at time \( t + \Delta t \), is given by the following equation:

\[ H_i(t + \Delta t) = H_i(t) + \frac{\Delta t}{6} \left( k_1 + 2k_2 + 2k_3 + k_4 \right) . \]  

5. RESULT ANALYSIS

In Table 1 [1], the symbols A, B, C and D represent the control rod banks followed by the number of steps withdrawn from the reactor core.

<table>
<thead>
<tr>
<th>Position of the control rod banks</th>
<th>Abscissa ( x )</th>
<th>( \rho ) (pcm)</th>
<th>( k )</th>
<th>1 - ( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A00B00C00D000</td>
<td>0.00</td>
<td>-4530.00</td>
<td>0.9566631</td>
<td>0.0433369</td>
</tr>
<tr>
<td>A10B00C00D000</td>
<td>5.00</td>
<td>-4131.45</td>
<td>0.9603247</td>
<td>0.0396753</td>
</tr>
<tr>
<td>A20B072C00D000</td>
<td>10.00</td>
<td>-3405.25</td>
<td>0.9670689</td>
<td>0.0329311</td>
</tr>
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<td>A225B172C044D000</td>
<td>15.10</td>
<td>-2587.80</td>
<td>0.9747747</td>
<td>0.0252253</td>
</tr>
<tr>
<td>A225B225C144D016</td>
<td>19.95</td>
<td>-1002.70</td>
<td>0.9900727</td>
<td>0.0099273</td>
</tr>
<tr>
<td>A225B225C225D116</td>
<td>25.05</td>
<td>-273.84</td>
<td>0.9972690</td>
<td>0.0027310</td>
</tr>
</tbody>
</table>

To insert the neutron counting \( M \) into the inverse point kinetic equation with external sources, we assumed that 100 steps of control rod banks will be withdrawn from the reactor core in 10 minutes of elapsed time. Table 2 shows the neutron counting values as function of time. These neutron counting values were obtained by using equation (9). We used a 4th-order polynomial function [7] to interpolate these neutron counting rates.

The kinetic parameters used in the inverse point kinetic equation [7] are: \( \Lambda = 0.00002 \text{s} \), \( \beta = 0.007 \), \( \beta_i = (0.000266, 0.001491, 0.001316, 0.002849, 0.000896, 0.000182) \), and \( \lambda_i (\text{s}^{-1}) = (0.0127, 0.0317, 0.115, 0.311, 1.4, 3.87) \) for 6 precursor groups.
Table 2. Subcritical counting rates

<table>
<thead>
<tr>
<th>t(s)</th>
<th>M(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23.075</td>
</tr>
<tr>
<td>600</td>
<td>25.204</td>
</tr>
<tr>
<td>1200</td>
<td>30.366</td>
</tr>
<tr>
<td>1800</td>
<td>39.643</td>
</tr>
<tr>
<td>2400</td>
<td>100.73</td>
</tr>
<tr>
<td>3000</td>
<td>366.16</td>
</tr>
</tbody>
</table>

Fig. 1 and Table 3 show the results obtained by the inverse point kinetic equation with external sources and compare the results with reactivity values in Table 1. In those calculations, we used neutron counting of Table 2, which were generated by a 4th-order polynomial function for a time step $\Delta t = 0.01s$.

Table 3. Result comparisons

<table>
<thead>
<tr>
<th>t(s)</th>
<th>$\rho (t)$ (pcm)</th>
<th>$\rho_{RK4} (t)$ (pcm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4530.00</td>
<td>-4530.00</td>
<td>0.00</td>
</tr>
<tr>
<td>600</td>
<td>-4131.45</td>
<td>-4180.10</td>
<td>1.18</td>
</tr>
<tr>
<td>1200</td>
<td>-3405.25</td>
<td>-3471.33</td>
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<tr>
<td>1800</td>
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<td>-2651.25</td>
<td>2.45</td>
</tr>
<tr>
<td>2400</td>
<td>-1002.70</td>
<td>-1025.34</td>
<td>2.26</td>
</tr>
<tr>
<td>3000</td>
<td>-273.84</td>
<td>-270.37</td>
<td>1.26</td>
</tr>
</tbody>
</table>

The results obtained from inverse point kinetic equation with external neutron sources are considered satisfactory when compared with the reactivity values of Table 1.

![Figure 1. Subcritical reactivity](image)
6. CONCLUSIONS

The objective of this paper was the development of a method to calculate reactivity, using the inverse point kinetic equation with external neutron sources.

The results obtained with the proposed method, have been found to be in good agreement with the reactivity values obtained for a fixed source benchmark calculation problem [1], for different configurations of control rod banks.

To use neutron counting rates obtained by the responses in ex-core neutron detectors in the inverse point kinetic equation with external sources it is necessary to make some corrections [2,7] in the detector signal.

REFERENCES