DANCOFF FACTORS WITH PARTIAL ABSORPTION IN CLUSTER GEOMETRY BY THE DIRECT METHOD

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ABSTRACT

Accurate analysis of resonance absorption in heterogeneous systems is essential in problems like criticality, breeding ratios and fuel depletion calculations. In compact arrays of fuel rods, resonance absorption is strongly affected by the Dancoff factor, defined in this study as the probability that a neutron emitted from the surface of a fuel element, enters another fuel element without any collision in the moderator or cladding.

In the original WIMS code, Black Dancoff factors were computed in cluster geometry by the collision probability method, for each one of the symmetrically distinct fuel pin positions in the cell. Recent improvements to the code include a new routine (PIJM) that was created to incorporate a more efficient scheme for computing the collision matrices. In that routine, each system region is considered individually, minimizing convergence problems and reducing the number of neutron track lines required in the in-plane integrations of the Bickley functions for any given accuracy.

In the present work, PIJM is extended to compute Grey Dancoff factors for two-dimensional cylindrical cells in cluster geometry. The effectiveness of the method is accessed by comparing Grey Dancoff factors as calculated by PIJM, with those available in the literature by the Monte Carlo method, for the irregular geometry of the Canadian CANDU37 assembly. Dancoff factors at five symmetrically distinct fuel pin positions are found in very good agreement with the literature results.
1. INTRODUCTION

Resonance absorption in heterogeneous systems is usually evaluated by firstly considering the fuel rods isolated in the moderator. The presence of other rods is incorporated later by using the Dancoff factor, which in its original definition was interpreted as a reduction in the effective surface of the fuel due to the adjacent rods. The in-current of resonance neutrons into the fuel rod in this model is reduced as compared to the in-current into a single fuel rod in an infinite moderator (shadowing effect). Alternatively, the Dancoff factor can be understood as a correction in the fuel escape probability due to the presence of other fuel rods in the system. For perfectly absorbing rods, both definitions are equivalent [2].

It has been observed that inaccuracies in computing both Grey and Black Dancoff factors, i.e., for partially and perfectly absorbing fuel rods, can lead to considerable errors in criticality, breeding ratios and fuel depletion calculations [3]. For this reason, much effort has been made in the past decades to further improve their models, a task that has been accomplished in connection with the development of faster computers.

The objective of the present work is to extend PIJM to compute Grey Dancoff factors by the collision probability definition, in two-dimensional cylindrical cell in cluster geometry, by the direct method for the CANDU37 irregular geometry.

2. DANCOFF FACTORS

Consider an infinitely long cylindrical cell divided into concentric annuli about the origin, each one independently subdivided into radial sectors, characterized by leading angles about a fixed direction, measured in an anticlockwise order, as shown in Fig.1. Onto this system, an array of rods is superimposed, each one independently divided radially and azimuthally about their centers. The azimuthal divisions, which may be different for each annulus of a rod, are characterized by leading angles about the outward line from the cell center to the rod center, measured also in an anticlockwise order.

Figure 1. Cylindrical cell in cluster geometry.
The regions thus formed are numbered from the innermost to the outermost cell annulus and for increasing values of the sectors’ leading angles. Rods inside an annulus are numbered in an anticlockwise order about the reference direction, after the sectors of annulus containing the rods have been numbered, and before moving to the next annulus. Regions of a rod are numbered in a way similar to those of the cell, with the difference that the sectors’ leading angles are now measured relative to the outward line from the cell center to the rod center. Voids are not numbered. In the case illustrated in Fig.1, the second and third annuli regions are subdivided into two sectors each, forming regions number 2 to 5. In addition, in this example, there are two types of superimposed rods, regions number 6 and 7, the last one with a cladding composed of two sectors, regions 8 and 9.

For perfectly absorbing fuel rods, the Dancoff factor can be defined by the relative reduction in the in-current of resonance neutrons into the fuel rod as compared to the in-current into a single fuel rod in an infinite moderator, as

\[ C = 1 - \frac{I}{I_0} = \int \frac{2\pi}{0} d\xi \int_{-\tau}^{\tau} K_i[\tau(\xi, y)]dy \]  

(1)

where \( r \) is the rod radius, \( \tau \) is the optical distance between perfect absorbers along the neutron track line, \( y \) and \( \xi \) are, respectively, the linear and azimuthal coordinates of the neutron track line, represented by its nearest distance to the rod center, and \( Ki_3 \) are Bickley functions of order three, given by

\[ Ki_3(x) = \int_0^{\pi/2} \sin^2 t \exp \left( -\frac{x}{\sin t} \right) dt. \]  

(2)

Alternatively, the Dancoff factor to a fuel rod \( i \) can be expressed using the concept of collision probabilities [5], as

\[ C_i = \frac{\left( \sum_j P_{ij} \right) - P_{ii}'}{1 - P_{ii}'} \]  

(3)

where \( P_{ij} \) is the probability of a neutron, escaping isotropically from rod \( i \), make its next collision in a fuel \( j \), \( P_{ii}' \) is the self collision probability in rod \( i \), and differs from \( P_{ii} \) for this last is the collision probability in any rod type \( i \), and where the sum is extended over all types of fuel in the cell. As example, the Dancoff factor of a fuel rod in region 6, in the hypothetic cell of Fig.1, is written as

\[ C_6 = \frac{P_{61} + (P_{66} - P_{66}') + P_{67}}{1 - P_{66}'} \]  

(4)

For partially absorbing rods, Dancoff factors will be computed by the above method.
3. COMPARISONS BETWEEN MONTE CARLO AND DIRECT METHOD

The Canadian cell CANDU37 is an AECL project consisting of 37 claddad pins, surrounded by the pressure tube, the gas gap and the calandria tube, as shown in Fig. 2. The coolant and the moderator are of D$_2$O, the sheath material is of zircaloy-4 and the fuel pins are all of the same size.

In order to compare the results with Monte Carlo based calculations, the PIJM routine of WIMS was adapted to compute Grey Dancoff factors from equation 3 and the same input parameters were used in our WIMS runs as those used in the Monte Carlo simulations made by Kim and Cho [1]. Fuel and cladding total macroscopic cross sections of 0.20 cm$^{-1}$ and 0.25 cm$^{-1}$, respectively, have thus been used for comparison with that work.

Grey Dancoff factors at five symmetrically distinct pin positions of CANDU37 core as shown in Fig. 2, for several values of the coolant cross-section, are presented in Table 1. Comparison with Dancoff factors as obtained by Kim and Cho [1], Fig. 3, show fairly good agreement between the results. In the most practical cases of irregular cells, comparisons with literature results are restricted to the verification of Dancoff factors behavior because these factors are usually available only in the graphic mode. The computational time was around 25 minutes in our simulations, against almost 4 hours with the Monte Carlo method.
Table 1. Grey Dancoff factors at five symmetrically distinct pin positions of CANDU37.

<table>
<thead>
<tr>
<th>$\Sigma_C$</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.414381</td>
<td>0.405885</td>
<td>0.355087</td>
<td>0.260213</td>
<td>0.259792</td>
</tr>
<tr>
<td>0.2</td>
<td>0.338953</td>
<td>0.321007</td>
<td>0.279161</td>
<td>0.196386</td>
<td>0.195026</td>
</tr>
<tr>
<td>0.4</td>
<td>0.286101</td>
<td>0.262709</td>
<td>0.227914</td>
<td>0.156655</td>
<td>0.154338</td>
</tr>
<tr>
<td>0.6</td>
<td>0.246358</td>
<td>0.220092</td>
<td>0.190686</td>
<td>0.129495</td>
<td>0.126331</td>
</tr>
<tr>
<td>0.8</td>
<td>0.215225</td>
<td>0.187734</td>
<td>0.162404</td>
<td>0.164712</td>
<td>0.105914</td>
</tr>
<tr>
<td>1.0</td>
<td>0.190122</td>
<td>0.162466</td>
<td>0.140215</td>
<td>0.0948676</td>
<td>0.0903928</td>
</tr>
<tr>
<td>1.2</td>
<td>0.169428</td>
<td>0.142285</td>
<td>0.122373</td>
<td>0.0831675</td>
<td>0.0782078</td>
</tr>
</tbody>
</table>

Figure 3. The cross section of the CANDU 37 element fuel bundle. The numbers 1, 2, 3, 4, and 5 marks symmetrically different pin positions [1].

By increasing artificially the fuel total cross section we obtain the Black Dancoff factors. It can be seen from Table 2 that the relative differences between the WIMS results and the Monte Carlo results were in the range 0.03% to 1.32%. The same relative differences are obtained if we compare the results using Eq. 3 with literature results using Eq. 1 [4], confirming that for Black Dancoff factors, both definitions are equivalent [2].
Table 2. Comparisons between Black Dancoff factors.

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct method</td>
<td>0.7855</td>
<td>0.7709</td>
<td>0.7359</td>
<td>0.4558</td>
<td>0.4526</td>
</tr>
<tr>
<td>Monte Carlo [1]</td>
<td>0.7863</td>
<td>0.7711</td>
<td>0.7263</td>
<td>0.4536</td>
<td>0.4529</td>
</tr>
<tr>
<td>Relative difference (%)</td>
<td>0.102</td>
<td>0.026</td>
<td>1.322</td>
<td>0.485</td>
<td>0.066</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

In this work, an analytical method for computing Grey Dancoff factors in cluster fuel cells with cylindrical outer boundary is proposed for the WIMS code. The method, based in the concept of collision probabilities, was incorporated to the PIJM module of WIMS. The method has been validated by comparison with Monte Carlo calculations in the more general case of AECL fuel bundle design of CANDU 37. The accuracy of the results, together with the decrease in the computation time, make the method recommended for routine calculations of Grey Dancoff factors in cluster fuel cells with cylindrical boundaries.

The work continues towards the extension of this calculation to fuel cells with square boundaries.

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