PET Reconstruction with System Matrix Derived from Point Source Measurements

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Abstract--The quality of images reconstructed by statistical iterative methods depends on an accurate model of the relationship between image space and projection space through the system matrix. A method of acquiring the system matrix on the CPS Innovations the HiRez scanner was developed. The system matrix was derived by positioning the point source in the scanner field of view and processing the response in projection space. Such responses include geometrical and detection physics components of the system matrix. The response is parameterized to correct point source location and to smooth projection noise. Special attention was paid to span concepts of HiRez scanner. The projection operator for iterative reconstruction was constructed, taking into account estimated response parameters. The computer generated and acquired data were used to compare reconstruction obtained by the HiRez standard software and produced by better modeling. Results showed that the better resolution and noise property can be achieved.

I. INTRODUCTION

The quality of reconstructed images that use algebraic methods depends on an accurate model of relationships between image and projection spaces. The elements of a system are commonly computed using simple geometric models, such as line integral, and assuming perfect detection. More complicated geometrical models take into account linear attenuation and inter crystal penetration in the detector response [1,2]. Monte Carlo simulations can further enrich system matrix models by incorporating detection physics effects such as inter crystal scatter [3,4]. Eventually, any model should be verified and corrected by measurements on a real scanner. Then the system matrix can be derived directly from these measurements. In this work, we exploit one aspect of the system matrix, the spatially variant point spread function (PSF), derived from measurements taken with the CPS HiRez scanner. Images, obtained by standard HiRez algebraic software and by a better system matrix modeling are presented. Both computer generated data and acquired data were used in this study.

II. METHOD

A. The HiRez scanner

Figure 1 represents a schematic view of the CPS Innovations HiRez scanner. The scanner consists of three rings of 48 LSO blocks. Each block represents 13x13 crystals with a crystal size of 4x4x20 mm. This scanner has a barrel shape, so that the radius of each crystal ring depends on the axial coordinate. Comparing with a cylindrical scanner, the HiRez has no more axial translation symmetry and the size of the stored system matrix significantly increases. There are also gaps between the blocks in radial and axial direction, which are assumed to be equal to the size of one crystal.

The projection data are sorted with a maximum ring difference of 27 in span 11, which produced a 336x336x313 sinogram, organized in 5 segments (copolar bin). Every plane with index ζ is a sinogram in line-of-response (LOR) space, parameterized by a radial coordinate ρ and an azimuthal angle θ. The radial bin size is 2 mm at the center of the field of view (FOV). The axial bin size is also 2 mm. Due to axial compression (span) each plane is a combination of five or six (individual/ original) axial LORs. Images were reconstructed on a 336x336x81 grid with 2 mm isotropic voxel.

B. System matrix acquisition tool: a 3D robot

A $^{68}$Ge point source (PS) with a diameter of 0.5 mm and an activity of 100 µCi was used in the measurements. Figure 2 shows the positioning device (3D robot). The robot allowed moving the point source along three orthogonal axes with a minimum step of 0.01mm. The robot was first aligned with respect to the scanner axes. The point source was placed inside a sector (blue area on Figure 1), corresponding to the symmetry of one block in the transverse plane. The PS was located on coarse grid of 1 cm. Axially, the PS locations covered half of the FOV with a step of 2 mm. Additional measurements along x and y axis were acquired in a couple of planes with fine sampling. The data were acquired for five minute per source position. The sinograms consisted of unscattered true events. The standard normalization procedure was applied to the data.

C. PSF modeling

Ideally the PS data can be used directly as a system matrix. Practically, the projection response needs to be modeled with some approximation because of imperfection in PS and scanner block locations, limited number of source positions and projection noise. First we ignored block edge effects (by excluding projection data too close from block edges) when estimating the parameters of our model described below. We also assumed the PSF model was independent of θ and segment. A separation of radial and axial response components was further assumed and each component was modeled differently.

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The PS data was acquired on a rather coarse grid in transverse plane. Then a fixed location PS was used to model the radial component, so the radial component is a function of radial sinogram coordinate for a given PS location. In this case we can model the scanner ring as continuous media sampled by crystals. The radial component is an asymmetrical function with a peak of response in projection space \( \rho_0 \) and combined from two half Gaussian functions. “Left” and “right” Gaussian functions differ in standard deviations.

The discrete nature of the crystals was taken into account in the modeling of the axial component, due to the span concept. We found that the normalization affected each sinogram plane differently and the use of a fixed PS did not produce a smooth function to be modeled. Instead we used the moving location PS data. The axial component was a superposition of symmetrical Gaussian responses. Each LOR was modeled according to the scanner geometry with a known depth of interaction (DOI). The DOI value was derived from a set of radial components with zero depth.

In summary, the general PSF function for given \( r, d \) and \( \zeta \) can be expressed as

\[
PSF(r, z) = A \exp\left\{ \frac{(r - \rho_0)^2}{2\sigma^2_{\rho}(\rho)} \right\} \sum_{i=1}^{\text{LOR in span}} \exp\left\{ \frac{(z - z_i - z_0)^2}{2\sigma^2_z} \right\},
\]

where

\[
\sigma_{\rho}(\rho) = \begin{cases} 
\sigma_{\text{left}}, & \rho < \rho_0 \\
\sigma_{\text{right}}, & \rho \geq \rho_0
\end{cases}
\]

The parameters \( A, \rho_0, \sigma_{\rho}, (\sigma_{\text{left}}, \text{ and } \sigma_{\text{right}}), z_0, \text{ and } \sigma_z \) were estimated by the Levenberg-Marquardt algorithm [3]. They were further regularized by polynomial functions and the knowledge of the scanner geometry.

Figure 3 and 4 display an example of PSF fitted to measured data. Figure 3(a) shows the radial component at two locations of the PS. The larger the radial distance, the lower and wider the response. The response has a larger tail toward the center of the FOV because of crystal penetration. Figure 4(b) shows the axial component, when the PS location has a significant depth. Then the response has a trapezoidal shape in a regular plane. A more complicated shape is observed in planes, which contain gaps between blocks and “missing” LORs.

Figure 3(b) shows the procedure to estimate the DOI. A group of measured points on radial axis with zero depth provides the estimation of \( \rho_0 \) as function of radial position \( r \). Taking into account the scanner geometry and penetration model, represented in insert of figure 3(b), the DOI was estimated to about 13 mm, which is larger then expected for LSO. However, we derived the DOI from response peak location, rather than from its center of mass. Parameter \( \rho_0 \) was estimated with that DOI during the calculation of the system matrix. Figure 3(c) shows \( \sigma_{\rho} \) estimation as function of \( r \). Left and right \( \sigma \) are regularized by a second degree polynomial with additional constraint of reflection symmetry with respect to \( r=0 \). The coefficients of polynomial regularization are used in the system matrix calculation. Figure 3(d) shows the estimated magnitude of response as function of \( r \). We enforced the constraint that the radial component should be normalized. The solid curve represent the magnitude, derived from the regularized \( \sigma_{\rho} \). There is some disagreements with experimental data at the edge of FOV. This magnitude derived from \( \sigma_{\rho} \) is used in the system matrix calculation.

### D. Forward projection implementation

The HiRez scanner has no axial translation symmetry and the storage of the system matrix requires large memory, even when all transverse plane symmetries are taking into account. We decided to use a pixel driven approach and stored the system matrix components for one angle only at fine grid of pixel locations. The radial component is function of radial and axial pixel position, but was assumed to be depth independent. This required about 1 Mb memory space for segment zero. The axial component is radial, axial and depth dependent, so its storage requirement was about 100 Mb for segment zero. It was assumed that the point source represents 2 mm pixel well.

We reconstruct images with OS-EM algorithm [6] with various weighting schemes [7]. The backprojector was matched with the forward projector. The reconstruction with...
PSF modeled system matrix will be referred as PSF reconstruction.

The HiRez standard algebraic reconstruction tool is also based on OS-EM. Sinogram first undergoes arc correction in both radial and axial direction to produce parallel beam (PB) sinogram. The forward projection uses linear interpolation in the approximation of the line integral. The reconstruction obtained by the standard reconstruction tool will be referred as PB reconstruction.

III. RESULTS

The main objective was the investigation of the benefits of better modeling with respect to our standard reconstruction software. The results presented in this report are reconstruction from segment zero (2D in case of PB reconstruction) data.

A. Computer Simulations

Computer simulations do show what kind of improvement can be achieved in an ideal situation, where the system matrix is known exactly. Data were generated by the PSF forward projector and Poisson noise was added, when necessary.

A resolution phantom with Gaussian objects arranged on a triangular grid, is shown on Figure 5(a). The 6 sectors contained objects with 2, 4, 6, 8, 10, 12 mm FWHM, respectively. The distance between object centers was fixed to 4°FWM. In the axial direction we choose a square function with a 4 planes periodicity: two planes are full and two planes are empty. PB reconstruction from noise free data shows a blur in all sectors for all transaxial planes. Moreover, the shapes are slightly elongated toward the FOV center (probably due to PSF asymmetry). PSF reconstruction has restored the correct shape in all sectors except in the 2mm one where the algorithm converges extremely slowly. A similar trend can be observed in the coronal view. PSF reconstruction can restore the 4 mm high frequency axial structure, which is not the case for the PB reconstruction. We observed that with PSF reconstruction it was nearly impossible to restore the 2mm high frequency axial structure (not shown here), probably because the span concept.

Noisy data were generated in case of 40 cm diameter cylinder with three hot and one cold axially uniform spots. Figure 5(b) plots the noise in a background ROI versus residual activity in the cold spot for a set of reconstruction with various iterations. The PSF reconstruction provided significantly superior noise property in comparison to PB reconstruction.

B. Experimental data

The resolution properties were evaluated on a phantom, combining of few PS acquired independently. Neither scatter nor attenuation correction was necessary. The transaxial view contains points located 1 cm apart. The axial structure was periodic: two planes with/without the PS. PB reconstruction shows a blurred image, while PSF reconstruction shows obviously an improved resolution in both radial and axial direction as can be observed in Figure 6(a).

The noise property was checked on a 20 cm diameter, 0.4 mCi sphere phantom shown in Figure 6(b). Data were acquired in true mode for four hours. Segment zero data contains 192,000 counts and estimated scatter fraction was 30%. Attenuation and scatter corrections for PSF reconstruction were obtained by applying inverse arc correction on the PB components. Attenuation weighted OS-EM algorithm was used. The cold spheres were used as cold spot, similarly to simulations. PSF reconstruction provided superior noise properties, consistently with the computer simulations.

IV. CONCLUSIONS

A method for acquiring and modeling the elements of the system matrix was developed for the HiRez clinical scanner. Special attention was paid to span concepts. The PSF modeling with some approximations readily provided superior reconstruction with respect to clinically used reconstruction software. The method allows further complexity in the PSF model by incorporating block edge effect. Better PSF modeling adds indeed complexity and is an expensive approach (time and memory-wise), when compared to the line integral model. However its noise reduction property can likely be beneficial in clinical environment especially at low statistics.

V. REFERENCES

Fig. 3. Examples of PSF radial component estimation. (a) Response from PS with $d=0$ and various $r$. (b) PSF peak $\rho_0$ provides estimation of DOI; (c) Estimation of “left” and “right” $\sigma_r$; (d) magnitude $A$ as function of PS radial distances. Radial component is shown for central axial plane.

Fig. 4. Examples of PSF axial component estimation. (a) Principles of depth dependence modeling of axial component. (b) Response from moving PS at $d=250\text{mm}$ and $r=0\text{mm}$. In plane 65, 6 axial LORs are grouped together; in plane 54 (a gap plane), only 4 axial LORs are grouped together.
Fig. 5 Computer Simulations. (a) Resolution phantom reconstruction (transaxial and sagittal view) from noise free generated data along with profiles through images. (b) Cylindrical contrast phantom reconstruction from noisy data along with plot of figures of merit.

Fig. 6 Experimental data. (a) Resolution phantom reconstruction (transaxial and coronal view) along with profiles through images. (b) Sphere phantom reconstruction along with plot of figures of merit.