Radiated Emissions from Printed Circuit Board Traces Including the Effect of Vias, as a Function of Source, Termination and Board Characteristics

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Abstract: The radiation from a printed circuit board (PCB) trace (microstrip) is analyzed including end effects. The end effects are due to the “vias” which connect the trace to the circuit board ground. The PCB trace currents are calculated from closed-form transmission line equations which includes dielectric effects of the board. The board ground plane is assumed to be infinite. Electric field magnitudes are then found from these currents using a free space Green’s function ignoring the board dielectric. This is compared with full-wave Method of Moments (MoM) analysis for finite size ground planes. The effect of ground plane size variations is noted. The worst case electric field magnitude is calculated from 50 to 2000 Mhz at numerous points on a sphere with a 3-meter radius. The via radiation is isolated from the PCB trace radiation to observe the relative magnitudes. The motivation for this work is twofold. We wish to quantify the magnitude of radiation, which occurs from differential mode currents flowing through horizontal components such as PCB traces and vertical components such as vias, and package leads. Also, we wish to see how this radiation is affected by source, termination and board characteristics.

INTRODUCTION

The literature indicates that much effort has been expended to model and predict the radiation from differential currents in PCB traces. In addition, it is a well known theory that differential mode currents induce ground plane “partial voltage” drops, which drive common mode currents. Common mode currents aside, we have not seen useful calculations which predict radiation from differential mode currents for simple structures. In this paper we use simple transmission line theory and a free space Green’s function to derive a closed-form solution for the radiation. We then compare our results with a full-wave MoM solution. Thus we attempt to show the effect of ignoring the PCB dielectric in computing the electric field but not in computing the transmission line currents. We also attempt to model combinations of PCB traces and vias and predict their relative contributions to the radiated fields. In addition, we show the relative emissions for PCB traces versus physical variables available to the circuit board designer. These are source, termination, and characteristic impedance as well as board thickness, line length and dielectric constant. The questions we try to answer are as follows. Do significant emissions occur due to differential mode currents flowing in microstrip structures on a typical digital PCB? If so, are there any cost effective means to reduce these emissions without the need to “bury” these lines between power and ground plane; that is, can any of the physical variables available to the circuit board designer be used to reduce these emissions? How much do the emissions from “vertical” structures such as vias (or package leads) contribute to the overall emissions? Finally, how accurate are simple closed-form solutions when compared with full-wave MoM analysis?

GEOMETRY

The structures analyzed in this paper are of the form shown in Figure 1. This is a slab of dielectric, with an infinitesimally thin PEC trace running along the top surface and a ground plane on the back surface. At each end of the line is a vertical z-directed strip (via) connected to the ground plane at one end and the trace at the other. For cases involving ideal transmission line theory, the ground plane is assumed to be infinite. In the MoM analysis, the ground plane and dielectric dimensions are as shown in Figure 1.

Figure 1 Simple transmission line structure with finite size ground plane.

The origin of the coordinate system is at the center of the transmission line in the plane of the ground. This is also the center of the sphere on which we calculate the worst case emissions.

1 Perfect Electric Conductor
fields later in this paper. A lumped source and source impedance is located on the negative x-axis at the position of the center of the z-directed strip (via). A lumped termination impedance is located on the positive x-axis, at the center of that via. The source is a delta-gap source in the MoM analysis. For all cases in this paper, the source strength is 1 volt and the source impedance is 10 Ohms, and the termination impedance varies.

FORMULATION

The formulation called “ideal” in this paper is based on elementary transmission line theory and ideal dipole theory, and as such, will only be briefly outlined here. The current on an ideal transmission line is given by

\[ I(x', f) = \frac{V_p(f)}{Z_C} \left[ e^{-jB(f)(x' - \frac{d}{2})} - \Gamma(f) e^{-jB(f)(x' - \frac{d}{2})} \right], \tag{1} \]

where \( V_p(f) \) is

\[ V_p(f) = V_s e^{jB(f)d} \left( 1 + \frac{Z_L}{Z_C} \right) + e^{-jB(f)d} \Gamma(f) \left( 1 - \frac{Z_L}{Z_C} \right), \tag{2} \]

and

\[ \Gamma(f) = \frac{Z_L(f) - Z_C}{Z_L(f) + Z_C}. \tag{3} \]

In the above equations, \( V_s \) is the source strength, \( d \) is the line length, \( Z_C \) is the characteristic impedance of the line, \( Z_s \) is the source impedance, \( Z_L \) is the (possibly complex) load impedance, \( B = 2nf/v_p \) is the propagation constant, \( x' \) is the distance along the line (with the origin at the center of the line), and \( f \) is the frequency. The characteristic impedance and the propagation velocity of the (waves on the) line are obtained using a quasi-static field solver. Equation 1, 2 and 3, give the current along the line from \(-d/2\) to \(d/2\) as a function of frequency. The x-directed current calculated at each end of the line is assumed to have the same magnitude as the z-directed current in the via. The length of the via is therefore ignored and the via is assumed to be short enough that the current is uniform over its length. The ground plane is assumed to be infinite and is removed by imaging the currents about the \( z=0 \) plane. Finally the electric field at 3 meters is calculated from these currents and their images using the previously mentioned Green’s function for the electric field[4]. This is (for \( \hat{z} \) directed currents)

\[ \mathbf{E}(r) = \hat{R} \frac{1}{j\omega E} \mathbf{R} \cdot \hat{z} e^{-jB\mathbf{R} \cdot \hat{z}} \left( \frac{jk}{R} + \frac{1}{R^2} \right) \]

\[ \hat{R} \times \left( \mathbf{R} \times \hat{z} \right) \frac{1}{j\omega E} \mathbf{R} \cdot \hat{z} e^{-jB\mathbf{R} \cdot \hat{z}} \left( \frac{k^2}{R} \frac{jk}{R} \frac{1}{R^2} \right) \tag{4} \]

where,

\[ \mathbf{R} = r - r'. \tag{5} \]

In equation (5), \( \mathbf{R} \) is the usual vector from the source point at \( r' \) to the field point at \( r \). Equation (4) is the expression for the electric field due to a point current source at \( r' \) and is integrated against the current (equation 1) to get the total field at \( r \). For this integration, the currents and their images are assumed to flow in a filament along the center of the lines and vias. Later in this paper, we account for real sources, which are assumed to have a trapezoidal, periodic shape. As mentioned, our source strength (in the frequency domain) is 1 volt. Real sources are accounted for by post multiplying our electric field magnitudes by Fourier amplitudes for an assumed trapezoidal waveform (that is, a time domain waveform). These amplitudes are given by

\[ V(n) = V_p \frac{T}{n^2} \left( \frac{\sin(\pi n trif)}{t_f} + \frac{\sin(\pi n trif)}{t_f^2} \right) \]

\[ -2\pi \frac{\sin(\pi n trif)}{t_f} \cos \left[ \frac{\pi n trif}{T} \right] \frac{1}{T} \]

where \( V_p \) is the amplitude of the assumed pulse train, \( T \) is the period, \( t_r \) is the rise time, \( t_f \) is the fall time, and \( t_d \) is the duty cycle time. For plots later in this paper, we use 5V, 10 ns, 1 ns, 1 ns, and 5.1' ns, respectively, for these quantities.

RADIATION FROM A TRACE WITH INFINITE GROUND PLANE

First, we compare the results using ideal transmission line theory and full-wave MoM analysis to gauge the accuracy of this approach. For this analysis, our trace over ground plane is assumed to be in free space. This is referred to below as “air dielectric”. In the ideal formulation, we calculate the currents from equation (1) and the electric field from equation (4). The electric field is calculated at multiple points on a 3-meter sphere and the worst case is plotted vs. frequency. Both analyses use image theory to eliminate the infinite ground plane. In the ideal transmission line theory, the currents are calculated everywhere along the transmission line. At each end of the line, the current is assumed to flow vertically (z-directed) through the vias. Then the currents are imaged to account for the infinite ground plane. Finally, the 3-meter fields are calculated using the standard free space Green’s function for the electric field[4]. This is (for \( \hat{z} \) directed currents)
Worst case electric field (3m) dBµV/m

The above graph shows the worst case electric field, calculated on a 3-meter sphere and plotted as worst case vs. frequency, for d=5.0 cm., h=.2 cm., and w=.1 cm. The ground plane is infinite (image theory is used). The characteristic impedance and propagation velocity is 166.9 Ω and 1.0 c0, respectively.

FINITE GROUND PLANE EFFECTS

Next we compare a MoM full-wave analysis for finite sized ground planes with the results obtained using ideal transmission line theory for infinite size ground planes. This allows us to gauge the effect of using finite size ground planes in the MoM analysis. The geometry is similar to the previous case except that the ground plane is finite with W=7.5 cm. and D=15.0 cm. As before, the dielectric is assumed to be air. The characteristic impedance and the propagation velocity is assumed to be the same as they were in the previous (infinite ground plane) case. That is, they are assumed not to be affected by the finite size of the ground plane. Also, the source and termination impedance are the same as in the previous case (10 Ω and 50 Ω). Figure 3 and Figure 4 show the impedance as seen by the source as a function of frequency. As expected, the impedance is 50Ω (real) at low frequency. The dashed lines are the results using ideal transmission line theory. The solid lines are the results obtained using MoM analysis. In the MoM analysis, the current is obtained for the geometry shown in Figure 1 which includes the effect of the vias. In the ideal transmission line theory, the current is obtained for a line of length d, and a characteristic impedance of 2Z0, without end effects. Currents through the vias are accounted for by extrapolating the currents at the ends of the line obtained by transmission line theory. In other words, the z-directed currents through the vias are assumed to be constant and equal in magnitude to the x-directed currents at the ends of the line. This will have an effect on the electric field calculation. The vias are completely ignored, however, in the calculation of the impedance. It is believed that the discrepancies evident in Figure 3 and Figure 4 are due to the fact that the parasitics from the vias are not included in the ideal transmission line results.

Figure 2. Magnitude of electric field, calculated on a 3-meter sphere and plotted as worst case vs. frequency, for d=5.0 cm., h=.2 cm., and w=.1 cm. The ground plane is infinite (image theory is used). The characteristic impedance and propagation velocity is 166.9 Ω and 1.0 c0, respectively.

Figure 3. Real part of input impedance for d=5.0 cm., h=.2 cm., and w=.1 cm. The ground plane size is W=7.5 cm., D=15.0 cm. in the MoM analysis. The characteristic impedance and propagation velocity is 166.9 Ω and 1.0 c0, respectively.

Figure 4. Imaginary part of input impedance for d=5.0 cm., h=.2 cm., and w=.1 cm. The ground plane size is W=7.5 cm., D=15.0 cm. in the MoM analysis. The characteristic impedance and propagation velocity is 166.9 Ω and 1.0 c0, respectively.

Figure 5 shows the worst case magnitude of electric field on a 3-meter sphere, calculated as previously described. The two curves labeled “Ideal” (dashed) are obtained from transmission line theory. The upper (dashed) curve is for a trace over an infinite ground plane (same as Figure 2). The lower dashed curve is for a trace over a “ground plane” with a width equal to the trace width. This represents a very narrow “ground return”. The two dashed curves are exactly 6 dB apart at all frequencies, as one might expect. The three other curves have varying ground plane sizes as follows: “Small GP” has W=1.0 cm., D=6.0 cm., “Med GP” has W=7.5 cm., D=15.0 cm., “Large GP” has W=30.0 cm., D=60.0 cm. Interestingly, we note that all finite size ground plane cases behave as the ideal narrow ground plane case at low enough frequencies and as the ideal infinite ground plane case at high enough frequencies. We also note that significant resonances can be observed in the “Med GP” and “Large GP” cases. These resonances correspond to the larger dimension of the ground plane.

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resonance in the “Large GP” case can be seen near 200 MHz. A resonance in the “Med GP” case occurs at about 800 MHz. This corresponds to half-wavelengths around 75 cm. and 19 cm., respectively. These are roughly the dimensions of the ground planes.

**Figure 5.** Magnitude of worst case electric fields calculated on a 3-meter sphere. The two curves labeled “Ideal” is calculated from transmission line theory. The other three curves are obtained from MoM analysis for finite size ground planes.

**MICROSTRIP RADIATION**

Next we examine the radiation from a PCB trace for a board with dielectric constant equal to 4. Again, we compare the results from ideal transmission line theory with a full-wave MoM analysis. Here we have used the dimensions d=6.0 in., h=7 mils, and w=6 mils. For the MoM analysis, we have used a finite ground plane size of W=300 mils and D=6.1 inches. Source and termination impedance's are once again 10 Ω and 50Ω, respectively. The characteristic impedance and propagation velocity is 79.7 Ω and .6 c0, respectively. Figure 6 and Figure 7 show the input impedance as seen by the source and Figure 8 shows the electric field magnitudes. Once again we show the ideal infinite ground plane and narrow ground plane cases, which are 6 dB apart. We note that the MoM solution shows better agreement with the ideal narrow ground plane case, as we would expect in light of the discussion in the previous section. That is, the ground plane must be very wide (which it is not, in this case) before the infinite ground plane case is approximated. We also note three resonances in the ideal case corresponding with minimums in the input impedance plots (see Figure 6 and Figure 7) as we would expect. It is not know at the present time what accounts for the difference between the ideal narrow ground plane analysis and the MoM solution. A ground plane resonance could be masking effect of the transmission line resonance in the MoM solution. Clearly the MoM solution has a more complicated shape with a different resonant behavior. Further simulations for this case are needed to ascertain the behavior of the MoM solution shown in Figure 8.
REALISTIC SOURCES AND ISOLATED TRACE RADIATION

Now we will examine the radiation characteristics from a digital PCB trace using the ideal transmission line theory. The dimensions are d=12.0 inch., h=7 mils, and w=6 mils. Also, in this case, we have assumed a trace thickness of 2 mils. The dielectric constant is again assumed to be 4. The characteristic impedance and propagation velocity is 72.5 Ω and .62 c0, respectively. The characteristic impedance and propagation velocity is 79.7 Ω and .6 c0, respectively. “Ideal_f” refers to the ideal case with a ground width equal to the strip width. “Ideal_i” refers to the ideal case with an infinite ground plane.

Figure 8. Electric field magnitude for microstrip with d=6.0 in., h=7 mils, and w=6 mils. The ground plane size is W=300 mils, D=6.1 in. in the MoM analysis. The characteristic impedance and propagation velocity is 79.7 Ω and .6 c0, respectively. “Ideal_f” refers to the ideal case with a ground width equal to the strip width. “Ideal_i” refers to the ideal case with an infinite ground plane.

Figure 9. Electric field magnitude for microstrip with d=12.0 in., h=7 mils, and w=6 mils. The characteristic impedance and propagation velocity is 72.5 Ω and .62 c0, respectively. The line is terminated in its characteristic impedance. The bar graph is the Fourier weighed results.

Figure 10. Figure 9 is repeated here (upper graph). The radiation from the PCB trace alone (without the vias) is shown in the other two plots.
CONCLUSIONS

The radiation from differential currents in PCB traces has been analyzed using simple, closed-form solutions. Excellent agreement with a full-wave MoM solution has been shown for the case of air dielectric. For realistic dielectrics, agreement is only fair. The dielectric appears to suppress the radiation significantly at some frequencies as shown by the MoM solution. Via radiation is a significant contributor to the overall radiation from circuit boards. In fact, it may dominate the radiation in some cases. Logical extension would indicate that radiation from package leads or any other structure causing z-directed currents may be overwhelmingly significant. We also have shown that ground plane resonances may enhance PCB radiation. Further study is needed. Full-wave MoM or other techniques are needed for this since realistic dielectrics may suppress the radiation significantly.

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REFERENCES