PREDICTING THE MAGNETIC FIELD FROM TWISTED THREE-PHASE ARRANGEMENT

Per Pettersson **Niclas Schönborg**
Vattenfall Utveckling AB
S-162 87 Stockholm, Sweden

Abstract: The low-frequency magnetic field in the vicinity of a twisted three-conductor arrangement carrying balanced three-phase current is studied by analytical methods. Formulas are derived for the field vector as function of observation point location, with pitch and radius of the helix as parameters. Exact analytical expressions are given for the field as function of time as well as for the effective value of the field components and the total field in form of Bessel-function series. The field vector is expressed in helical coordinates. Approximations for the field far away from the arrangement are given. Verifying measurements have been performed on a rig.

INTRODUCTION

Twisting of pair-wire lines in telephone cables is known to be an effective way to suppress inductive crosstalk among the lines. This method utilizes two effects of twisting: Immunity of any line to the fields from the other lines, and minimization of the field emitted by each of the lines. It is the latter effect which will be the object of the paper. Another application of twist is to reduce stray fields from power leads in e.g. space probes and aeroplanes. As a utility, our interest in the field reduction option offered is caused by the controversial question of magnetic fields and human health. Potential power industry applications would be generator and substation busbars, though twisted transmission lines seem impracticable. The method is directly applicable on cables, of course. Three-phase current is predominantly used in the power industry which means that the three-phase, three-wire line will correspond to the one-phase, pair-wire line in communications.

The object of the paper is to present a theory of the compound magnetic field from three infinitely long and helically shaped current filaments carrying balanced three-phase current. Excepting [2-4] of these authors, only one paper [1] treating the three-phase case has been found in the literature and just a few treating the one-phase case [5-9]. Our main reference will be H. Buchholz [5,6] who did the pioneering work as early as 1937. The main vehicle will be his expression for the field from a single helix. The field of the one- and three-phase arrangements are then obtained by superposition. The contribution of the later research lies mainly in producing simple engineering approximations, either by direct ways or by using first-term approximations to series expansions in Bessel functions. Compared to earlier studies, this paper will use the natural, here called helical, coordinates for the field vector by which one component will be eliminated and the problem thereby be rendered two-dimensional. This very special property of the field was noted already in [5] but was missed in many later works.

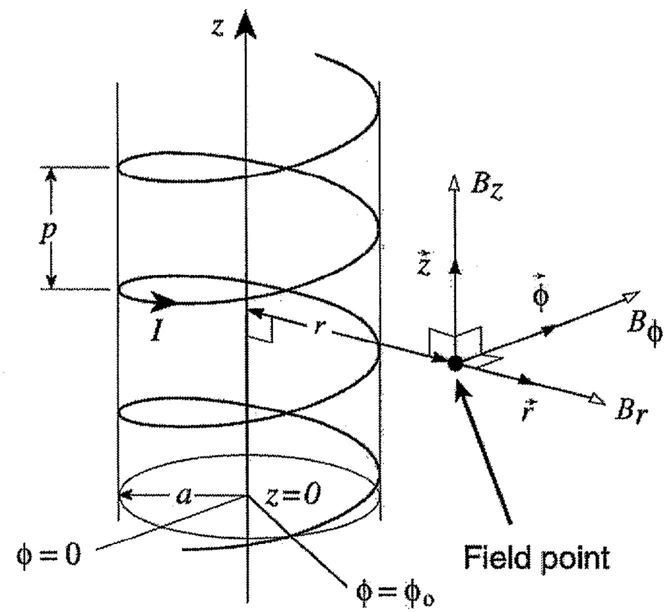

Figure 1. Helical line current

THE EXACT THEORY

Fig. 1 shows the basic one-helic case. Here I is current, a is radius and p is pitch. Cylinder coordinates are used, designated by r, ϕ and z for radial, azimuth angle and axial coordinate, respectively. By [5,6] the radial, azimuthal and axial field components are

$$B_r = \frac{\mu_0 Ia}{\pi r^2}(kr)^2 \sum_{n=1}^{\infty} nI_n'(nka)K_n'(nkr)sin\left[n(\phi-\phi_0-kz)\right] \quad (1)$$

$$B_\phi = \frac{\mu_0 I}{2\pi r}$$
$$+\frac{\mu_0 Ia}{\pi r^2}(kr)\sum_{n=1}^{\infty} nI_n'(nka)K_n(nkr)cos\left[n(\phi-\phi_0-kz)\right]$$

0-7803-4140-6/97/$10.00

$$B_z = -\frac{\mu_0 I a}{\pi r^2}(kr)^2 \sum_{n=1}^{\infty} n I_n'(nka) K_n(nkr) \cos[n(\phi-\phi_0-kz)]$$

with $k = \frac{2\pi}{p}$

Here $I_n(x)$ and $K_n(x)$ are the modified Bessel functions of first and second kind of order n, and $I_n'(x)$ and $K_n'(x)$ their derivatives. The problem has recently been revisited in [10]. Eq. (1) holds for $r>a$ and similar equations apply to the case $r<a$, i.e. inside the cylinder.

It is observed that, neglecting the single term of B_ϕ, we have $B_z = -(kr)B_\phi$ which simply means that the field component in the ϕz-plane is perpendicular to an imagined helix of pitch p through the field point, see Figure 2. Then the field can be described by only two components, namely the radial component and the so called binormal component B_b, since the tangential component B_s is zero. The binormal direction $\vec{b}$ is indicated in Fig. 2 along with the tangential direction $\vec{s}$, which together make the lateral surface coordinates for the field. The third direction is the so called normal direction, which, but for an opposite sign, agrees with the radial direction. We choose to retain the radial component B_r. The relation between the two reference systems is given by

$$B_s = B_z \sin\psi + B_\phi \cos\psi$$
$$B_b = B_z \cos\psi - B_\phi \sin\psi \qquad (2)$$

with $\psi = \tan^{-1}(kr)$

Here ψ is the pitch angle of the field-point helix.

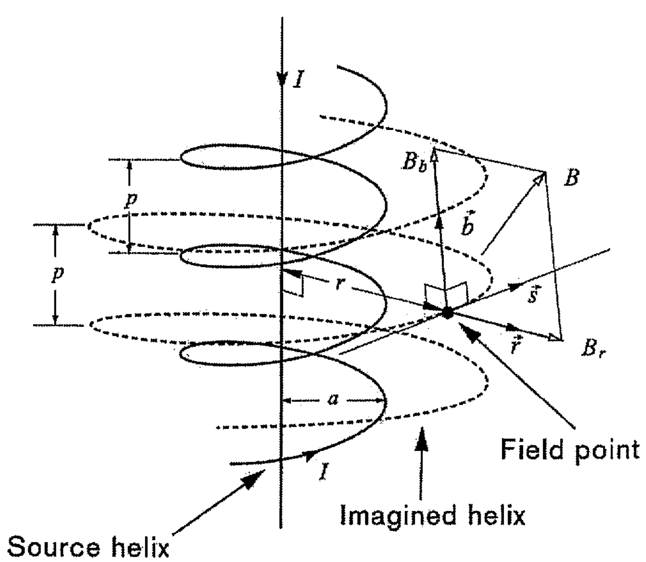

Figure 2. Field components in helical coordinates

Now, cancelling the single term of B_ϕ, by applying an imagined return current in the cylinder axis, we can write for the time dependent field-vector components

$$B_r = \hat{B}_0 \gamma^2 \sum_{n=1}^{\infty} n I_n'(n\eta) K_n'(n\gamma) \sin n\Phi \sin \omega t \qquad (3)$$

$$B_b = -\hat{B}_0 \gamma \sqrt{1+\gamma^2} \sum_{n=1}^{\infty} n I_n'(n\eta) K_n(n\gamma) \cos n\Phi \sin \omega t$$

$$\hat{B}_0 = \frac{\mu_0 \hat{I} a}{\pi r^2} \qquad \eta = ka \qquad \gamma = kr \qquad \Phi = \phi - \phi_0 - kz$$

where time dependence $I = \hat{I} \sin \omega t$ is assumed, with ω denoting angular frequency and t time. Here $\hat{B}_0$ is introduced for briefness in notations. Note that the cylindrical coordinate system is retained for the field-point location.

For the three-phase case of study, we number the wires by i=1,2,3 and set

$$I_i = \hat{I} \sin(\omega t + \alpha_i) \quad \alpha_i = (n-1)\frac{2\pi}{3} \quad \phi_i = (i-1)\frac{2\pi}{3} \qquad (4)$$

where α_i and ϕ_i are phase angles and location parameters of the three helices. This means that the conductor arrangement in a transverse plane will form an equilateral triangle since the three coaxial helices have equal radii. Term-wise addition of the three fields yields after some elementary calculations using the auxiliary geometric relations

$$\sum_{i=1}^{3} \sin(\omega t + \alpha_i) \sin[n(\phi - \phi_i - kz)] = \mp \frac{3}{2} \cos(\omega t \pm n\Phi) \qquad (5)$$
$$\sum_{i=1}^{3} \sin(\omega t + \alpha_i) \cos[n(\phi - \phi_i - kz)] = \frac{3}{2} \sin(\omega t \pm n\Phi)$$

the following results for the field components

$$B_r = \frac{3}{2} \hat{B}_0 \gamma^2 \sum_n (\mp n) I_n'(n\eta) K_n'(n\gamma) \cos(\omega t \pm n\Phi) \qquad (6)$$
$$B_b = -\frac{3}{2} \hat{B}_0 \gamma \sqrt{1+\gamma^2} \sum_n n I_n'(n\eta) K_n(n\gamma) \sin(\omega t \pm n\Phi)$$
$$\Phi = \phi - kz$$

The summations range over n=1,2,4,5,7… i.e. all positive integers except n=3,6,9…. The upper sign applies for n=2,5,8… and the lower for n=1,4,7…, and so the signs for the remaining terms will be alternating.

The effective values of the components and the total field B can now be established according to

$$B_r = \frac{3}{2} B_0 \gamma^2 \left\{ \sum_n \sum_m (\mp n)(\mp m) I_n'(n\eta) I_m'(m\eta) \right. \quad (7)$$

$$\left. K_n'(n\gamma) K_m'(m\gamma) cos(\pm n \mp m)\Phi \right\}^{1/2}$$

$$B_b = \frac{3}{2} B_0 \gamma \sqrt{1+\gamma^2} \left\{ \sum_n \sum_m nm I_n'(n\eta) I_m'(m\eta) \right.$$

$$\left. K_n(n\gamma) K_m(m\gamma) cos(\pm n \mp m)\Phi \right\}^{1/2}$$

$$B = \frac{3}{2} B_0 \gamma^2 \left\{ \sum_n \sum_m nm I_n'(n\eta) I_m'(m\eta) [(\mp 1)(\mp 1) K_n'(n\gamma) K_m'(m\gamma) \right.$$

$$\left. + \frac{1+\gamma^2}{\gamma^2} K_n(n\gamma) K_m(m\gamma)] cos(\pm n \mp m)\Phi \right\}^{1/2}$$

with $B_0 = \dfrac{\mu_0 I a}{\pi r^2}$

where the index and sign rules of above apply. I denotes the effective value of the current.

Formulae (7) can be applied to the special case with an untwisted configuration by letting η and γ tend to zero, since p tends to infinity, using the small argument approximations for the Bessel functions.

$$I_n(\eta) \approx \frac{\left(\frac{\eta}{2}\right)^n}{n!} \qquad K_n(\gamma) \approx \frac{1}{2} \frac{(n-1)!}{\left(\frac{\gamma}{2}\right)^n} \qquad \eta, \gamma << 1 \qquad (8)$$

The result is

$$B_r = \frac{3}{2} B_0 \left\{ \sum_n \sum_m \left(\frac{a}{r}\right)^{n+m-2} \frac{1}{4} (\mp 1)(\mp 1) cos(\pm n \mp m)\Phi \right\}^{1/2} \quad (9)$$

$$B_b = \frac{3}{2} B_0 \left\{ \sum_n \sum_m \left(\frac{a}{r}\right)^{n+m-2} \frac{1}{4} cos(\pm n \mp m)\Phi \right\}^{1/2}$$

$$B = \frac{3}{2} B_0 \left\{ \sum_n \sum_m \left(\frac{a}{r}\right)^{n+m-2} \frac{1}{4} [(\mp 1)(\mp 1) + 1] cos(\pm n \mp m)\Phi \right\}^{1/2}$$

THE APPROXIMATE THEORY

In certain cases the first terms of the above series expansions will be so dominant that they can serve as a good approximations for the whole series. One important such case is when the configuration has a loose twist in the meaning that $a<<p$ and the field point is distant in the meaning that $r>>p$. In such case the total field in (7), for example, reduces to

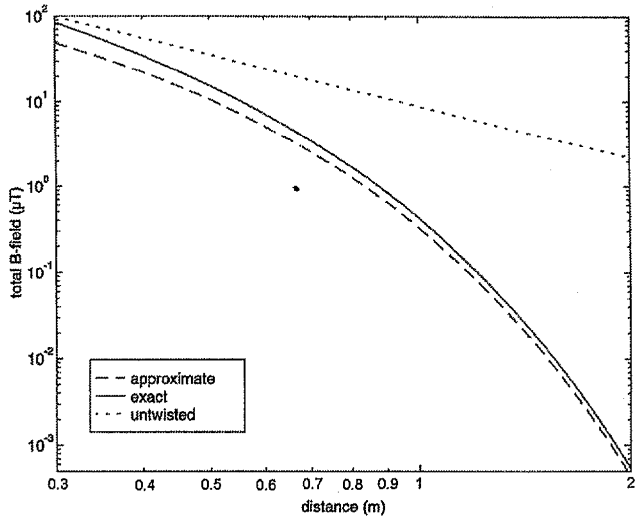

Figure 3. Approximate vs. exact total magnetic field

$$B \approx \frac{3}{2} B_0 \gamma^2 I_1'(\eta) \left[K_1'^2(\gamma) + \frac{1+\gamma^2}{\gamma^2} K_1^2(\gamma) \right]^{1/2} \qquad (10)$$

The corresponding expressions for the component fields are obvious. Still, three rather inaccessible Bessel functions are involved and further simplification is wanted. Using the small argument approximation $I_1'(\gamma) \approx 1/2$ by (8), and the large argument approximations

$$K_1(\gamma) \approx -K_1'(\gamma) \approx \sqrt{\frac{\pi}{2\gamma}} e^{-\gamma} \text{ for } \gamma >> 1 \qquad (11)$$

gives, from (10)

$$B \approx F \cdot \frac{3}{4} \sqrt{2} B_0 \qquad F = \sqrt{\frac{\pi}{2}} \gamma^{3/2} e^{-\gamma} \qquad (12)$$

Now, the first-term approximation for the untwisted case is from the third equation of (9) just $3/4 \cdot \sqrt{2} B_0$, which holds for distant field points, $r>>a$. Then (12) tells that the field reducing effect of twisting a straight arrangement is captured by F. F may be called "twist factor". The epithet is all the more justified since F in effect applies individually to both the r- and the b-component. It should moreover be stressed that the b-direction will tend to the z-direction for increasing distances, while it will constantly be the ϕ-direction in the straight case. In the limit, both cases will have a circular polarisation for the field vector in the respective planes.

Figure 3 demonstrates the precision of the approximation (12) for the total field for a sample case with a=0.1 m, p=1 m, and I=1 A. It appears that the approximation works very well even when γ is not overly large. As a rule of thumb, the error will be smaller then 10 % when $\gamma > 10$ as long as $\eta < 0.5$. The corresponding curve for the untwisted case is shown in the diagram to demonstrate this precision. 10 % relative precision is seen to be achieved at about 0.5 m.

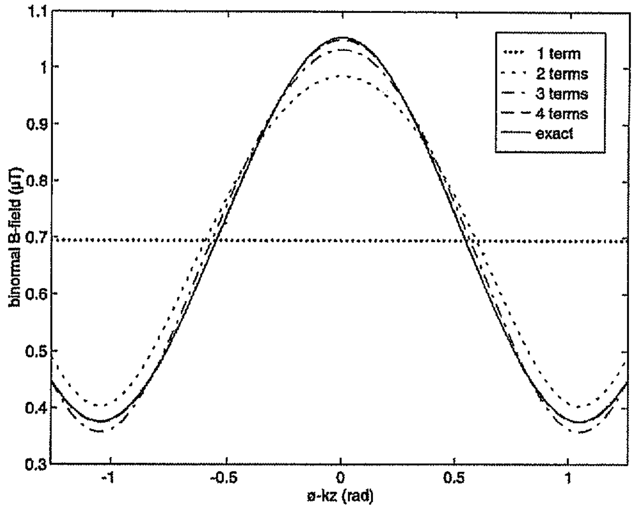

Figure 4. Convergence of binormal field component

Figure 4 is showing the convergence of the Bessel series for the binormal component of (7) at distance 0.2 m, with varying field-point location along the line for the sample case above. It is evident that the approximation works very well even this close, and it seems to give a good average value of the field along the line.

As an illustration of what field reduction that can be brought about by twist we see that for distance to pitch ratios, r/p, equal to 1 and 2, the twist factor, F, is equal to 0.037 and 0.00020, respectively. This shows the extreme fastness in field decay with distance.

VERIFICATIONS

The correctness of the exact analytical theory was verified in two different ways: By comparison with the results of a numerical simulation based on Biot-Savart's law, and by comparison with measurements on a laboratory rig. The data for the demonstration case are a = 0.1 m, p = 1m and I = 200 A. In this presentation we will focus on the experiment, and just say that the agreement between the numerical method and the analytical one was excellent up to many figures.

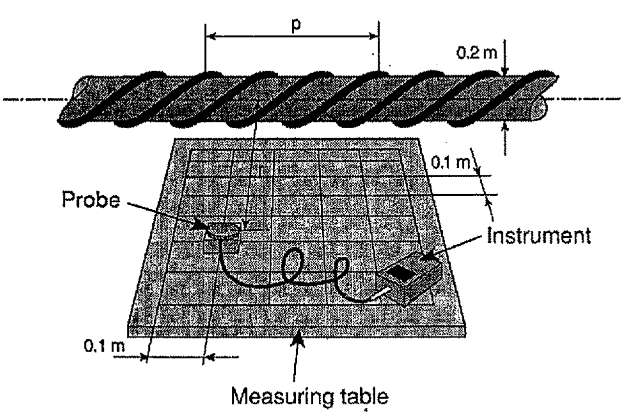

Figure 5. Set-up of experiment

Fig. 5 shows the experiment set-up. Three plastic coated 50 mm^2 stranded Cu-wires were helically wound on a plastic pipe and connected in one end and fed by a transformer with 200A, 50 Hz current in the other end. The length of the pipe was 11 m, which was considered enough to be simulating the infinite case for distances less than 2 m. Two pitch values were tested, 0.5 m and 1.0 m, of which only the latter will be reported here. The coil of the probe was mounted on a block of wood designed to keep the center of the coil in an horizontal plane through the axis of the pipe when measuring the three field components in cylindrical coordinates. For each distance, the field was measured at six points spaced 0.1 m on a centered measuring table, and the average value was computed. Conversion of the field components from cylindrical into helical coordinates was then performed by use of (2) after which the total field was calculated.

Fig. 6 shows the comparison between the measured and calculated total field. The agreement is seen to be excellent for distances up to about 1.0 m. The deviations for distances above were found to be caused by small accidental offsets of the windings during the erection of the rig so that they came to deviate from perfect helices.

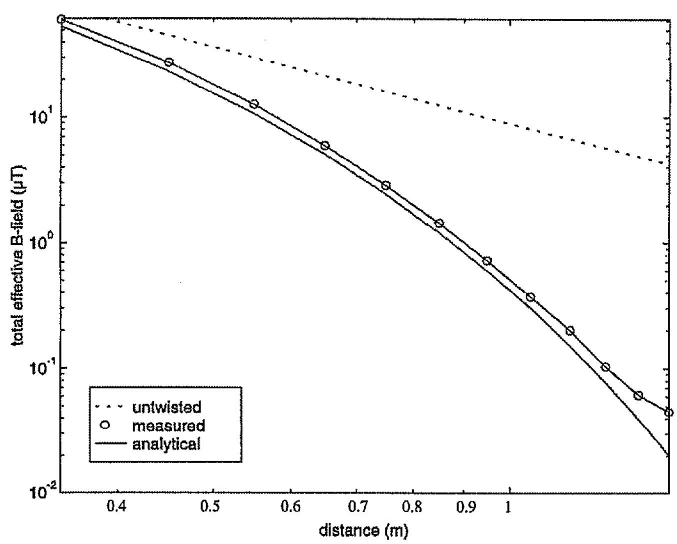

Figure 6. Theoretical vs. measured total magnetic field

Figure 7 shows the effect of this defect on the tangential component of the field. Remember that this field component is supposed to be non-existing, but is here present for the whole range of distances. The tangential component was in effect of comparable size with the radial and binormal components, not shown, for distances slightly above 1 m. Also shown in the diagram is the calculated untwisted reference case. It is seen that, despite of the defects, the field reduction is almost two decades for the largest distance shown (1.45 m).

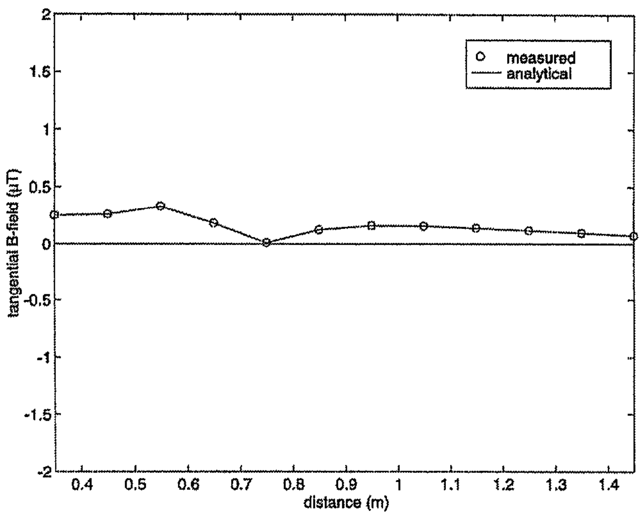

Figure 7. Theoretical vs. measured tangential component

FUTURE WORK

It is interesting to compare configuration twist with other field reducing options. One such is split of phases so that the current of each phase is distributed equally on two or more subphases placed symmetrically on a circle, see e.g. [2]. By this, the inverse power law exponent of r for the far field decay is raised from two for the three-wire case to become $n+1$, where n is the number of subphases. However, the twisted line will eventually outperform any phase-split line for large distances as it has a virtually exponential decay with r by (12). One interesting continuation of the work of this paper would then be study of a combination of the two methods, i.e. a twisted split-phase. This will lead to still lower fields. Such configurations for insulated cables can also be practically advantageous, since the smaller area of the cables would facilitate twisting.

A second research object would be shielded twisted arrangements. Also here, the foundations are well laid by Buchholz. Still, useful approximations for the remote field remains to be established.

CONCLUSIONS

A complete and exact theory of the power-frequency magnetic field emitted by a twisted three-phase configuration is presented. The theory includes formulae for the time varying field vector components at an arbitrary point in space, as well as formulae for the effective values of the vector components and the total field. The field vector has been expressed in helical coordinates, which eliminates one of three vector component. The theory predicts excellent low-field performance for twisted arrangements. The correctness of the analytical solution is demonstrated by comparison with experimental results.

Approximate expressions for the field at a distant point are established. The essence of twist in reducing the field is found to be captured by a certain "twist factor" of simple form, to be applied to the untwisted configuration. The precision of the approximation is demonstrated on a sample case to be very good even for not very large distances.

The paper revises earlier results appearing in the literature and presents new insight into this important problem. Further objects of research are identified.

REFERENCES

[1] Haber, F., "The Magnetic Field in the Vicinity of Parallel and Twisted Three-wire Cable Carrying Balanced Three-Phased Current", IEEE Trans. Electromagn. Compat., Vol. EMC-16, No. 2, pp. 76-82, May 1974

[2] Pettersson, P., "Principles in Power System Magnetic Field Reduction", Stockholm Power Tech, Paper SPT HV 12-01-0428, pp. 358-363, June 1995

[3] Pettersson, P., Schönborg, N., "The Magnetic Field from Twisted Three-Phase Arrangements", EMC'96 ROMA, Paper T-1, pp 834-839, Sept. 1996

[4] Pettersson, P., Schönborg, N., "Reduction of Power system Magnetic Field by Configuration Twist", IEEE PES Winter Meeting, Paper No. PE-141-PWRD-0-12-1996, Feb. 1997

[5] Buchholz, H., "Elektrische Strömungsfelder mit Schraubenstruktur", Elektrische Nachrichten-technik, pp. 264-280, 1937

[6] Buchholz, H., "Elektrische und Magnetische Potentialfelder", Chapter 6, Springer-Verlag, Berlin/ Göttingen/Heidelberg, 1957

[7] Alksne, A.Y., "Magnetic Fields near Twisted Wires", IEEE Trans. Space Electr. Telem., Vol. SET 10, pp. 154-158, Dec. 1964

[8] Moser, J.R., Spencer, R.F., "Predicting the Magnetic Field from a Twisted Pair Cable", IEEE Trans. Electromagn. Compat., Vol. EMC-10, pp. 324-329, Sept. 1968

[9] Shenfeld, S., "Magnetic Fields of Twisted-Wire Pairs", IEEE Trans. Electromagn. Compat., Vol. EMC-11, No. 4, pp. 164-169, Nov. 1969

[10] Hagel, R., Gong, L., Unbehauen, R., "On the Magnetic Field of an Infinitely Long Helical Line Current", IEEE Trans. Magn., Vol. 30, No. 1, pp. 80-84, Jan. 1994