On Calculation of Shield Effectiveness of Spherical Shell

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Abstract: Screening is one of necessary means for the protection of electrical and electronic systems against EMI. Here we give the method for calculating shielding effectiveness of N-layer spherical shells.

Introduction

The metallic shield is often used to reduce electromagnetic interference. According to different conditions the shield is made of magnetic material or diamagnetic material. In special cases, multi-layer shielding bodies consisting of diamagnetic and magnetic materials are used to increase shield effectiveness.

Now, much literature discusses the characteristics of the multi-layer shield (less than 3 layers). This article studies the screening problem of N-layer spherical shells. According to the electromagnetic field equations and boundary conditions, the matrix of screening factor and reflecting factors of N-layer spherical shells have been obtained. Applying this formula, one can directly get the intensity of electromagnetic field. This formula may be used for spherical shell with any number of layers and was verified with N=1, 2, 3, so its universality was confirmed. According to different requirement properly choosing the material of each layer, one can get satisfactory result.

Besides this, the permeability of shield material is a function of the magnetic field strength. As an example, in this work a significantly improved method of estimating the permeability of one-layer shield is presented. This method provides a basis for analysis of the shielding effectiveness of practical spherical shell.

Shield Effectiveness of a Multi-Layer Spherical Shell

In spherical coordinate system shown in Fig.1, the relationship between field components can be written as following:

\[
\frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (E_\theta \sin \theta) - \frac{\partial E_\phi}{\partial \phi} \right] = -j \omega \mu H_r,
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r E_r) - \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} = -j \omega \mu H_\theta,
\]

\[
\frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (H_r \sin \theta) - \frac{\partial H_\phi}{\partial \phi} \right] = -j \omega \mu H_\theta,
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) - \frac{1}{r} \frac{\partial E_r}{\partial \phi} = (\sigma + j \omega \epsilon) E_\phi.
\]

Assume there is a small horizontal loop perpendicular to z-axis in the \( \phi \)-plane, as shown in Fig.1, which carries AC current and acts as an interference source, then we have

a) field intensity distribution is independent of \( \phi \), i.e. \( \partial H/\partial \phi = 0 \) and \( \partial E/\partial \phi = 0 \),

b) \( E_r = 0, H_\phi = 0 \) and \( E_\phi = 0 \).

thus, the Maxwell's equations can be written as

\[
\frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (E_\theta \sin \theta) - \frac{\partial E_\phi}{\partial \phi} \right] = -j \omega \mu H_r,
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r E_r) - \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} = -j \omega \mu H_\theta,
\]

\[
\frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (H_r \sin \theta) - \frac{\partial H_\phi}{\partial \phi} \right] = -j \omega \mu H_\theta,
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) - \frac{1}{r} \frac{\partial E_r}{\partial \phi} = (\sigma + j \omega \epsilon) E_\phi.
\]

Substituting (1), (2) into (3), we can obtain

\[
\frac{\partial^2 E_\phi}{\partial \theta^2} + \frac{2}{r} \frac{\partial E_\phi}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{\cos \theta}{r^2} \frac{\partial E_\phi}{\partial \theta} - \frac{1}{r^2 \sin^2 \theta} E_\phi = k^2 E_\phi
\]

where \( k = \sqrt{j \omega \mu (\sigma + j \omega \epsilon)} \).

Let's discuss the field in the shield shell and in the dielectric, respectively.

1. field in the shield shell

\[
\frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (E_\theta \sin \theta) - \frac{\partial E_\phi}{\partial \phi} \right] = -j \omega \mu H_r,
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r E_r) - \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} = -j \omega \mu H_\theta,
\]

\[
\frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (H_r \sin \theta) - \frac{\partial H_\phi}{\partial \phi} \right] = -j \omega \mu H_\theta,
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) - \frac{1}{r} \frac{\partial E_r}{\partial \phi} = (\sigma + j \omega \epsilon) E_\phi.
\]

Solve equation (4) by the separation variable method, and consider the physical conception, we can obtain [Ref.1]

\[
E_\phi = \sum_{i=1}^{n} \{ C_i h_n^{(1)}(jkr) + D_i h_n^{(2)}(jkr) \} \sin \theta = E_i^m + E_i^r
\]

\[
H_\phi = \frac{1}{j \omega \mu} \sum_{i=1}^{n} \{ C_i \frac{\partial}{\partial r} [h_n^{(1)}(jkr)] + D_i \frac{\partial}{\partial r} [h_n^{(2)}(jkr)] \} \cos \theta = H_i^m + H_i^r
\]

where \( h_n^{(1)}(jkr) \) and \( h_n^{(2)}(jkr) \) are the first and second kind of
spherical Hanker functions, respectively, and $P_n^m(\cos \theta)$ is associated Legendre polynomial. The superscript "in" and "r" of the field intensity indicate incident wave and reflected wave, respectively.

for $|kr| >> 1$

$$h_n^{(1)}(jkr) = \frac{1}{jkr} e^{-kr\left(\frac{\cos \theta}{2}\right)}$$
$$h_n^{(2)}(jkr) = \frac{1}{jkr} e^{kr\left(\frac{\cos \theta}{2}\right)}$$

Obviously, the wave impedance of the metallic shield can be written as the following

$$Z_{in} = \frac{B_{in}}{E_{in}} = -\sqrt{\frac{j\omega \mu}{\sigma}} \quad \text{(for incident wave)}$$
$$Z_{r} = \frac{B_{r}}{E_{r}} = \sqrt{\frac{j\omega \mu}{\sigma}} \quad \text{(for reflected wave)}$$

2. Field in the dielectric

For quasi-steady state case, we have $\sigma + j\omega \approx 0$, so $k = 0$. Thus the solution of the equation (1) is as following [Ref.1]

$$E_{in} = \sum_{m=1}^{n} (A_m e^{-n\phi} + B_m e^{n\phi}) P_n^m(\cos \theta) = E_{in}^r + E_{in}^m$$
$$H_{in} = \frac{1}{j\omega \mu} \sum_{m=1}^{n} [A_m (n+1) e^{-n\phi} - B_m e^{n\phi}] P_n^m(\cos \theta) = H_{in}^r + H_{in}^m$$

the wave impedance of the dielectric can be written as

$$Z_{in}^d = \frac{E_{in}^d}{H_{in}^d} = -\sqrt{\frac{j\omega \mu}{\sigma}} \quad \text{(for incident wave)}$$
$$Z_{r}^d = \frac{E_{r}^d}{H_{r}^d} = \sqrt{\frac{j\omega \mu}{\sigma}} \quad \text{(for reflected wave)}$$

Let’s assume $r_0$ being the internal radius of the spherical shell, $t_1, t_2, \ldots, t_N$ being the thickness and $k_1, k_2, \ldots, k_N$ being the related parameters of each layer from inside to outside, respectively. Since tangential electric and magnetic field components continue at $r = r_0$, $r = r_0 + t_1$, $r = r_0 + t_1 + t_2$, $\ldots$, $r = r_0 + t_1 + t_2 + \ldots + t_N$ planes, we can get a set of equations, and then we can obtain the following equations

$$\begin{bmatrix} \begin{bmatrix} S \\ S_{in} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} m_1 + m_2, & \ldots, & m_N \\ m_1 + m_2, & \ldots, & m_N \\ \end{bmatrix} \begin{bmatrix} \text{ch}_{k_1} \text{l}_{t_1} & \ldots & \text{ch}_{k_N} \text{l}_{t_N} \\ \end{bmatrix} \begin{bmatrix} 1 - p \\ p \end{bmatrix} \begin{bmatrix} \text{ch}_{k_1} \text{l}_{t_1} \\ \end{bmatrix} \times \begin{bmatrix} \text{ch}_{k_1} \text{l}_{t_1} & \ldots & \text{ch}_{k_N} \text{l}_{t_N} \\ \end{bmatrix} \times \begin{bmatrix} \text{ch}_{k_1} \text{l}_{t_1} \\ \end{bmatrix} \begin{bmatrix} -N_1 \text{sh}_{k_1} \text{l}_{t_1} \\ \end{bmatrix} \begin{bmatrix} 1 \\ m_1 + m_2 \\ \end{bmatrix} \end{bmatrix}$$

where $S$ is the ratio of the field intensity passing through the shield to incident field intensity, and $P$ is the total reflection factor at $r = r_0$.

$$N_i = \frac{Z_{d_i}}{Z_{m_1}}$$
$$Z_{d_1} = m_1, \quad Z_{d_2} = m_2, \quad Z_1 = \frac{j\omega \mu \sigma}{n}, \quad Z_2 = \frac{j\omega \mu \sigma}{n+1}$$

Thus, we can get

$$S = \frac{(m_1 + m_2)(CB - AD)}{m_1 m_2 B - m_1 A + C - m_2 D} \quad \text{(5)}$$

for $N=1$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \text{ch}_{k_1} \text{l}_{t_1} & \ldots & \text{ch}_{k_N} \text{l}_{t_N} \\ \end{bmatrix} \begin{bmatrix} \text{ch}_{k_1} \text{l}_{t_1} & \ldots & \text{ch}_{k_N} \text{l}_{t_N} \\ \end{bmatrix} \times \begin{bmatrix} \text{ch}_{k_1} \text{l}_{t_1} \\ \end{bmatrix} \begin{bmatrix} 1 \\ m_1 + m_2 \\ \end{bmatrix} \begin{bmatrix} -N_1 \text{sh}_{k_1} \text{l}_{t_1} \\ \end{bmatrix} \begin{bmatrix} 1 \\ m_1 + m_2 \\ \end{bmatrix} \end{bmatrix}$$

i.e. $A = \text{ch}_{k_1} \text{l}_{t_1}$, $B = \frac{1}{N_1} \text{sh}_{k_1} \text{l}_{t_1}$, $C = -N_1 \text{sh}_{k_1} \text{l}_{t_1}$, $D = \text{ch}_{k_1} \text{l}_{t_1}$

substituting into (5), we have

$$S = \frac{1}{\frac{m_1 + m_2}{N_1} \text{sh}_{k_1} \text{l}_{t_1}}$$

for $N=2$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \text{ch}_{k_2} \text{l}_{t_2} & \ldots & \text{ch}_{k_N} \text{l}_{t_N} \\ \end{bmatrix} \begin{bmatrix} \text{ch}_{k_2} \text{l}_{t_2} & \ldots & \text{ch}_{k_N} \text{l}_{t_N} \\ \end{bmatrix} \times \begin{bmatrix} \text{ch}_{k_2} \text{l}_{t_2} \\ \end{bmatrix} \begin{bmatrix} 1 \\ m_1 + m_2 \\ \end{bmatrix} \begin{bmatrix} -N_1 \text{sh}_{k_2} \text{l}_{t_2} \\ \end{bmatrix} \begin{bmatrix} 1 \\ m_1 + m_2 \\ \end{bmatrix} \end{bmatrix}$$

i.e. $A = \text{ch}_{k_2} \text{l}_{t_2} \text{ch}_{k_1} \text{l}_{t_1} + \frac{N_1}{N_2} \text{sh}_{k_2} \text{l}_{t_2} \text{sh}_{k_1} \text{l}_{t_1}$

$$B = -\frac{1}{N_1} \text{ch}_{k_2} \text{l}_{t_2} \text{sh}_{k_1} \text{l}_{t_1} - \frac{1}{N_2} \text{sh}_{k_2} \text{l}_{t_2} \text{ch}_{k_1} \text{l}_{t_1}$$

$$C = -N_2 \text{sh}_{k_2} \text{l}_{t_2} \text{ch}_{k_1} \text{l}_{t_1} - N_1 \text{ch}_{k_2} \text{l}_{t_2} \text{sh}_{k_1} \text{l}_{t_1}$$

$$D = \frac{N_2}{N_1} \text{sh}_{k_2} \text{l}_{t_2} \text{ch}_{k_1} \text{l}_{t_1} + \text{ch}_{k_2} \text{l}_{t_2} \text{ch}_{k_1} \text{l}_{t_1}$$

substituting $A$, $B$, $C$ and $D$ into (5), we can obtain

$$S = \frac{1}{\frac{m_1 + m_2}{N_1} \text{sh}_{k_1} \text{l}_{t_1}} \quad \text{(7)}$$

In like manner, we can obtain for $N=3$

$$S = \frac{1}{\frac{m_1 + m_2 + m_3}{N_1} \text{sh}_{k_1} \text{l}_{t_1} \text{sh}_{k_2} \text{l}_{t_2} \text{ch}_{k_1} \text{l}_{t_1}} \quad \text{(7)}$$

where

$$T = 1 + F_1 \text{sh}_{k_1} \text{l}_{t_1} + F_2 \text{ch}_{k_1} \text{l}_{t_1} + F_3 \text{sh}_{k_1} \text{l}_{t_1} + F_4 \text{ch}_{k_1} \text{l}_{t_1} + F_5 \text{sh}_{k_1} \text{l}_{t_1} + F_6 \text{ch}_{k_1} \text{l}_{t_1} + F_7 \text{sh}_{k_1} \text{l}_{t_1} + F_8 \text{ch}_{k_1} \text{l}_{t_1}$$

$$F_1 = \frac{Z_{d_1} + Z_{d_2}^2}{Z_{m_2}(Z_1 + Z_2)}; \quad F_2 = \frac{Z_{d_2} + Z_{d_3}^2}{Z_{m_2}(Z_1 + Z_2)}; \quad F_3 = \frac{Z_2 + Z_{d_1}^2}{Z_{m_2}(Z_1 + Z_2)}; \quad F_4 = \frac{Z_1 + Z_{d_1}^2}{Z_{m_2}(Z_1 + Z_2)}; \quad F_5 = \frac{Z_2 + Z_{d_2}^2}{Z_{m_2}(Z_1 + Z_2)}; \quad F_6 = \frac{Z_1 + Z_{d_2}^2}{Z_{m_2}(Z_1 + Z_2)}; \quad F_7 = \frac{Z_2 + Z_{d_3}^2}{Z_{m_2}(Z_1 + Z_2)}; \quad F_8 = \frac{Z_1 + Z_{d_3}^2}{Z_{m_2}(Z_1 + Z_2)}.$$
Formulas (6), (7) and (8) is identical with Ref. 3 and 4, where the formulas are suitable for one-, two- and three-layer shield. However, formula (5) is of more universal significance because it is suitable for an arbitrary N-layer spherical shell. For high frequency case, we have \( \sigma + \omega = \omega \), thus \( Z_1 = Z_2 = Z_3 = \frac{\mu_0}{\varepsilon_0} \), and formula (5) will valid also.

The shielding attenuation can be calculated as the following:

\[
A_s = \ln \left| \frac{1}{S} \right| N_p = 8.68 \ln \left| \frac{1}{S} \right| \text{ dB}
\]

The Shield Effectiveness of Single-Layer Thin Spherical Shell under Static Magnetic Field

As shown in Fig.2, the outside interference source is a uniform magnetic field denoted by \( H_o \), the internal and external radii of the spherical shell are \( a \) and \( b \), respectively, and the thickness of the spherical shell is \( t \) (\( t \ll b \)).

It is well known that the shield effectiveness for this case can be written as

\[
S = \frac{1}{1 + \frac{\mu_r}{2 \mu_r} \left( \frac{1 - a^3}{b^3} \right)} \left( 1 + \frac{1}{3 \mu_r M} \right)
\]

where \( M = \frac{t}{b} = \frac{b-a}{b} \).

Since the value of \( \mu_r \) changes with the magnetic field \( H \), and \( H \) varies with angle \( \theta \), the value of \( \mu_r \) should be very well selected for calculation.

The magnetic field \( H \) inside shell varies with angle \( \theta \). When \( \theta = 0 \) or \( \theta = \pi/2 \), \( H \) reaches its maximum or minimum [Ref.5].

\[
H = H_{\text{max}} = \frac{H_0}{1 + \frac{2}{3} \mu_r M} \tag{10}
\]

\[
H = H_{\text{min}} = \frac{H_0}{\mu_r + 2 \mu_r^2 M} \tag{11}
\]

Using (10) and (11), we can be obtain

\[
A_s = \ln \left| \frac{1}{S} \right| N_p = 3.09 N_p = 26.5 \text{ dB}
\]

References


Fig.3

\[
\mu_r = \frac{3}{2M} \left( \frac{H_0}{H} - 1 \right)
\]

\[
\mu_r = \frac{3}{4M} \left( \frac{1}{1 + \frac{8H_0}{3H} - M - 1} \right)
\]

As an example, assuming there is a magnetic material of the characteristic curve (solid) as shown in Fig.3, and a spherical shell shield made of such material with \( M = 0.01 \). When \( H_0 = 1 \) Oersted, we can obtain two dotted curves in Fig.3 by using equations (12) and (13). The intersection value of the upper dotted curve and the solid curve is the maximum, \( H_{\text{max}} = 6000 \), and the intersection value of the lower dotted curve and the solid curve is the minimum \( H_{\text{min}} \). Therefore, \( \mu_r \) can be approximately determined by the average value of \( H_{\text{max}} \) and \( H_{\text{min}} \), i.e. \( \mu_r = 3000 \). Thus

\[
S = \frac{1}{1 + \frac{2}{3} \mu_r M} \approx \frac{1}{1 + \frac{2}{3} \times 3000 \times 0.01} = \frac{1}{21}
\]

the shield attenuation is

\[
A_s = \ln \left| \frac{1}{S} \right| N_p = 26.5 \text{ dB}
\]