Low-Frequency Shielding Effectiveness of Inhomogeneous Enclosures

P. Kistenmacher, A. Schwab
Institute of Electric Energy Systems and High-Voltage Technology
University of Karlsruhe, Germany

Abstract — This paper presents a two dimensional analytical solution for electromagnetic shielding problems at frequencies below resonances. The application of the method to three dimensional shields yields good approximations, except for errors nearby edges which are not parallel to the magnetic field. The solution is valid for enclosures of arbitrary shape with any given combination of materials and wall thicknesses. Considering skin effects, the shielding effectiveness is derived by solving the Helmholtz equation for individual parts of the wall with different thicknesses and materials. The solution for the total shield is then found by applying Faraday's law in integral form. Three cases are discussed: heterogeneous enclosures consisting of several wall sections with different materials and wall thicknesses, coated, laminated and nested shields and multi-cavity enclosures. Different examples of combinations of non-magnetic and magnetic materials are discussed in detail. Finally, the potential of saving material and weight by using multiple shields is investigated.

Low-Frequency Eddy Current Shielding (Inductive Shielding)

The problem of calculating the electromagnetic shielding effectiveness (SE) is yet unsatisfactory, because the common methods are limited to ideal shapes, e.g. a circular cylindrical shell or a sphere. The SE is defined as the magnetic field strength at a particular point with and without an enclosure, i.e. the insertion loss. Ideal shapes possess a homogeneous internal magnetic field amounting a locally independent SE. However, the field inside technical enclosures with their edges and apertures is inhomogeneous, consequently the SE depends on the location considered. Analytical solutions for those shapes do not exist. The present article pursues to derive the SE of complex geometries by means of permissible simplifications.

The usefulness of numerical methods is limited, because modeling of thin walls and field diffusion through conducting materials is numerically difficult. In contrast, the general analytical solution presented in this paper allows the calculation of the SE for a screen of arbitrary shape consisting of different materials and wall thicknesses with moderate efforts. It is valid for wavelengths greater than the dimension of the screen. In that case the shielding effect is exclusively provided by eddy currents in the shield's wall, Fig.1.

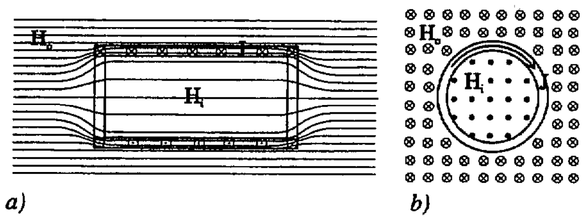


Figure 1: Shielding effectiveness of a cylindrical shield

Considering long wavelengths, it is justified to neglect the displacement current. The cross section parallel to the magnetic field , Fig.1a), causes only local effects, like the edge effect, where a higher magnetic field appears near the edges. The SE is mainly determined by the geometric shape of the cross-section perpendicular to the magnetic field, Fig.1b). In this plane, the eddy current is confined in the shield's wall and, hence, determines the magnetic field inside the enclosure according to Ampere's law. The displacement and refraction of the magnetic field at the walls due to magnetic materials is neglected in the following, because inductive shielding is the main shielding principle already at low frequencies. For these reasons the main interest lies in the investigation of an enclosure of arbitrary cross-section in a uniform external magnetic field parallel to the shield's walls, so-called longitudinal field [1], which reduces the problem to two dimensions. In this case the field inside is always homogeneous.

The described problem was already investigated by Thomson [2] in 1893, when he derived the SE of a hollow circular cylinder. King [3] presented an analytical solution of a hollow sphere and Cooley [4] for a rectangular enclosure. A comprehensive compilation of analytical shielding problems can be found in Kaden [1]. Finally, Latham and Lee [5] derived the SE of homogeneous enclosures of arbitrary shape.

Arbitrary Cross Section

The complex SE is defined as H_i/H_o, which considers as well the phase shift between the outer and inner magnetic field. It is derived by solving Maxwell's equations subject to appropriate boundary conditions. Below, the SE of an enclosure of arbitrary cross section, Fig.2, with a constant wall thickness d of homogeneous material is derived [6].

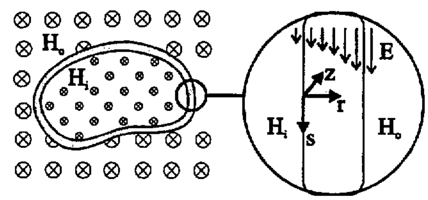


Figure 2: Enclosure of arbitrary cross section.

Inside and outside the magnetic fields are uniform H_i and H_o and parallel to the shield's walls, Fig.2. Therefore the determination of the electric and magnetic field inside the conducting walls are of foremost importance for solving the problem. The physics in the conductor is described by a Helmholtz equation.

$$\nabla^2 \boldsymbol{H} = k^2 \boldsymbol{H} \quad \text{with } k^2 = j\omega\mu_0\mu_r\sigma \tag{1}$$

In this case the magnetic field has only one spatial component, which is the component in the z-direction. The current density of the induced eddy current **J**, and consequently the electric field **E**, are perpendicular to the magnetic field **H** in the x,y-plane. For the calculations, a new coordinate system is advantageous, which is defined as an s-component along the perimeter and the r-component normal to the wall, Fig.2. On account of the restriction to frequencies below resonances, displacement currents can be neglected. Therefore, **J** and **E** are always oriented in s-direction parallel to the contour of the enclosure. A component of **J** in r-direction would require a displacement current. Further, we can assume that the variation of the electric and magnetic field along the s-direction can be neglected compared to the change in r-direction.

$$\frac{\partial \boldsymbol{H}}{\partial s} \ll \frac{\partial \boldsymbol{H}}{\partial r} \quad \text{and} \quad \frac{\partial \boldsymbol{E}}{\partial s} \ll \frac{\partial \boldsymbol{E}}{\partial r} \tag{2}$$

These legitimate simplifications reduce the vector partial differential equation (1) to a scalar differential equation.

0-7803-3207-5/96/$5.00 © 1996 IEEE

$$\frac{\partial^2 H_z(r)}{\partial r^2} = k^2 H_z(r) \quad (3)$$

The general solution of (3) is

$$H(r) = C_1 \cosh(kr) + C_2 \sinh(kr) \quad (4)$$

with the boundary values

$$\begin{aligned} H(0) &= H_i = C_1 \\ H(d) &= H_o = H_i \cosh(kd) + C_2 \sinh(kd) \end{aligned} \quad (5)$$

The constant C_2 is derived by Faraday's law in integral form

$$\oint \mathbf{E}ds = -\iint j\omega\mu\mathbf{H}d\mathbf{A} \quad (6)$$

for which the electric field has to be determined. Due to the neglection of displacement currents, the electric field along the inner boundary is constant and can be derived by means of Ampere's law.

$$\begin{aligned} &\nabla \times \mathbf{H} = \sigma \mathbf{E} \\ &E_s(r) = -\frac{1}{\sigma}\frac{\partial H_z(r)}{\partial r} \\ &E_s(r) = -\frac{H_i k}{\sigma}\sinh(kr) - \frac{C_2 k}{\sigma}\cosh(kr) \\ &E_s(0) = -\frac{C_2 k}{\sigma} \end{aligned} \quad (7)$$

Using this result in (6) along the inner boundary leads to the determining equation for the constant C_2

$$\begin{aligned} &-\frac{C_2 k}{\sigma} \cdot perimeter = -j\omega\mu_0 H_i \cdot cross\ sectional\ area \\ &C_2 = \frac{k}{\mu_r} H_i \frac{cross\ sectional\ area}{perimeter} \end{aligned} \quad (8)$$

This finally results in the solution of the SE of an enclosure of arbitrary cross section.

$$\boxed{\frac{H_i}{H_o} = \frac{1}{\cosh(kd) + \dfrac{k}{\mu_r}\dfrac{cross\ sectional\ area}{perimeter}\sinh(kd)}} \quad (9)$$

Using the geometric parameters of a plate and cylinder shield we obtain the well-known solutions of [1]. The ratio of the cross sectional area to the perimeter is obviously the essential quantity determining the SE, which results simply from Faraday's law of induction. A uneven contour leads to a poorer SE than a smooth one, due to the longer perimeter.

ENCLOSURE WITH DIFFERENT MATERIALS AND WALL THICKNESSES (HETEROGENEOUS ENCLOSURE)

In order to account for the possibility of different wall properties and wall thicknesses, an individual solution of the Helmholtz equation (1) must be found separately for each homogeneous part of the wall. A small thickness compared to the length of a homogeneous part is required $d_n \ll l_n$, but this is always true for technical enclosures. As before, only the plane of the eddy current, normal to the magnetic field, is considered, i.e. a two-dimensional problem. The example in Fig.3 is a rectangular enclosure, where part 1 consists of material 1 with the thickness d_1 and the bottom, part 2, is made out of material 2 of the thickness d_2.

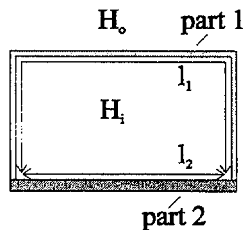


Figure 3: Shield with different wall properties

For each part a separate general solution (10) of the Helmholtz equation must be chosen.

part 1: $H_1(r) = C_{11}\cosh(k_1 r) + C_{21}\sinh(k_1 r)$ (10)

part 2: $H_2(r) = C_{12}\cosh(k_2 r) + C_{22}\sinh(k_2 r)$

with $k_1^2 = j\omega\mu_0\mu_{r1}\sigma_1$ and $k_2^2 = j\omega\mu_0\mu_{r2}\sigma_2$

Due to a homogeneous inner and outer magnetic field the boundary values are

$$\begin{aligned} H(0) &= H_i = C_{11} = C_{12} \\ H(d_1) &= H_o = H_i\cosh(k_1 d_1) + C_{21}\sinh(k_1 d_1) \\ H(d_2) &= H_o = H_i\cosh(k_2 d_2) + C_{22}\sinh(k_2 d_2) \end{aligned} \quad (11)$$

Like above, the determination of the constants C_{2n} requires Faraday's law and, thereby, the electric field, see (7), which is derived by means of Ampere's law(12).

$$\begin{aligned} &\nabla \times \mathbf{H} = \sigma \mathbf{E} \\ &E_{s1}(0) = -\frac{C_{21}k_1}{\sigma_1} \\ &E_{s2}(0) = -\frac{C_{22}k_2}{\sigma_2} \end{aligned} \quad (12)$$

Substituting (12) for the electric field in Faraday's law and integrating along the inner boundary of the enclosure yields

$$\begin{aligned} &\oint_{inner\ perimeter} \mathbf{E}ds = -\iint_{inner\ area} j\omega\mu\mathbf{H}d\mathbf{A} \\ &-\frac{C_{21}k_1}{\sigma_1}\cdot l_1 - \frac{C_{22}k_2}{\sigma_2}\cdot l_2 = -j\omega\mu_0 H_i \cdot cross\ sectional\ area \end{aligned} \quad (13)$$

Eliminating the constants C_{21} and C_{22} by virtue of (12) yields the SE of the enclosure of Fig.3.

$$\frac{H_i}{H_o} = \frac{\dfrac{k_1 l_1}{\sigma_1\sinh(k_1 d_1)} + \dfrac{k_2 l_2}{\sigma_2\sinh(k_2 d_2)}}{j\omega\mu_0 \cdot csa + \dfrac{k_1 l_1}{\sigma_1\tanh(k_1 d_1)} + \dfrac{k_2 l_2}{\sigma_2\tanh(k_2 d_2)}} \quad (14)$$

In general, for an enclosure consisting of N different wall sections we obtain

$$\boxed{\begin{aligned} &\frac{H_i}{H_o} = \frac{\sum_1^N \dfrac{k_n l_n}{\sigma_n\sinh(k_n d_n)}}{j\omega\mu_0 \cdot cross\ sectional\ area + \sum_1^N \dfrac{k_n l_n}{\sigma_n\tanh(k_n d_n)}} \\ &\text{with} \quad \sum_1^N l_n = perimeter \end{aligned}} \quad (15)$$

Obviously, the weakest part with the least conductivity or the smallest wall thickness determines the SE. A gap parallel to the magnetic field, corresponding to a zero conductivity results in a SE of 1, i.e., 0 dB.

As an example, the SE of a quadratic enclosure with a bottom plate of different conductivity σ_2 similar to Fig.3 will be calculated. The enclosure's wall is made of copper ($\sigma_1=5.8\cdot10^7$ S/m) with a thickness of 1mm, the side length is 1m. Fig.4 shows the SE depending on the conductivity of the bottom plate.

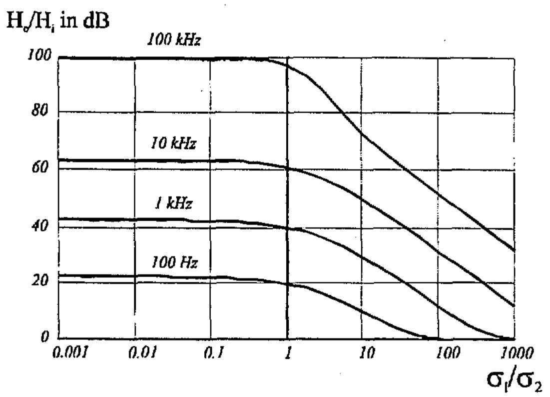


Figure 4: Shielding effectiveness of a quadratic shield with different wall properties, depending on the conductivity σ_2 of the bottom

If the conductivity of the bottom plate is much higher than the conductivity of the top, the SE remains constant, because the eddy current is limited by the conductivity of the top. It obviously does not make sense to use a better material. If the conductivity of the bottom is less than that of the top, then the eddy current is limited in the bottom and the SE decreases.

COATED SHIELDS

Coatings are mainly used to improve the SE of plastic shields [7]. For special purposes multilayered materials are employed. The thickness of the layers ranges from some µm to a mm. Sometimes it is assumed that special combinations of materials and thicknesses exist which especially improve the SE, like coated glasses in optics. At first glance, the application of Schelkunoff's method or of the wave matrix method would seem the appropriate tool to derive the solution [8]. These methods are very advantageous, because they are suitable also aat high frequencies and consider resonances Unfortunately, they are limited to one dimensional problems. Therefore we use the same method as before.

The solution is derived for a shield consisting of several layers depicted in Fig.5. Its layers are described by a thickness d_n, a conductivity σ_n and a relative permeability μ_m. The magnetic field is orientated parallel to the walls Therefore only the two-dimensional longitudinal field problem needs to be solved.

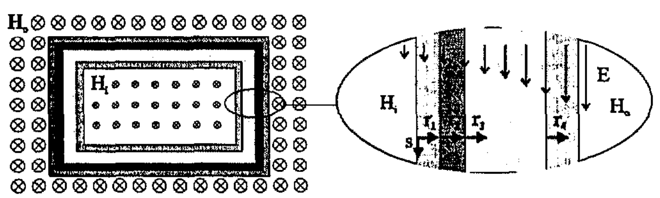


Figure 5: Multi-layer enclosure

As before, the Helmholtz equation (1) needs to be solved, but now separately for each layer. This means the behavior of the magnetic field is different in each layer which requires for each one a separate general solution. For a wall of N layers this results in

$$\begin{aligned} H_1(r_1) &= C_{11}\cosh(k_1 r_1) + C_{21}\sinh(k_1 r_1) \\ H_2(r_2) &= C_{12}\cosh(k_2 r_2) + C_{22}\sinh(k_2 r_2) \\ &\vdots \\ H_N(r_N) &= C_{1N}\cosh(k_N r_N) + C_{2N}\sinh(k_N r_N) \end{aligned} \tag{16}$$

There are only two boundary conditions

$$\begin{aligned} H_1(0) &= H_i \\ H_N(d_N) &= H_o \end{aligned} \tag{17}$$

but the continuity of the magnetic and electric field (18) must be considered as well.

$$\begin{aligned} H_{n-1}(d_{n-1}) &= H_n(0) \\ E_{sn-1}(d_{n-1}) &= E_{sn}(0) \end{aligned} \tag{18}$$

The continuity of the magnetic field leads to

$$C_{1n-1}\cosh(k_{n-1}d_{n-1}) + C_{2n-1}\sinh(k_{n-1}d_{n-1}) = C_{1n} \tag{19}$$

while the continuity of the electric field results in

$$\begin{aligned} E_{sn}(r) &= -\frac{1}{\sigma_n}\frac{\partial H_n(r)}{\partial r} = -\frac{k_n}{\sigma_n}\left(C_{1n}\sinh(k_n d_n) + C_{2n}\cosh(k_n d_n)\right) \\ \frac{k_{n-1}}{\sigma_{n-1}}&\left(C_{1n-1}\sinh(k_{n-1}d_{n-1}) + C_{2n-1}\cosh(k_{n-1}d_{n-1})\right) = \frac{k_n}{\sigma_n}C_{2n} \end{aligned} \tag{20}$$

Obviously, the constants for one layer depend on the constants of the previous layer. Therefore, the constants C_{11} and C_{21} are of foremost importance, because then all other constants can be calculated.

From (17) follows

$$C_{11} = H_i \tag{21}$$

For the determination of C_{21}, the electric field and Faraday's law are employed in the same way as already presented for the enclosure of arbitrary cross section (8).

$$\begin{aligned} \oint_{\text{inner perimeter}} \mathbf{E}ds &= -\iint_{\text{inner area}} j\omega\mu\mathbf{H}dA \\ C_{21} &= \frac{k}{\mu_r}H_i\frac{\text{cross sectional area}}{\text{perimeter}} \end{aligned} \tag{22}$$

The reciprocal dependency of the constants is best described by matrices,

$$\begin{bmatrix} C_{1n+1} \\ C_{2n+1} \end{bmatrix} = \begin{bmatrix} \cosh(k_n d_n) & \sinh(k_n d_n) \\ \frac{\sigma_{n+1}}{\sigma_n}\frac{k_n}{k_{n+1}}\sinh(k_n d_n) & \frac{\sigma_{n+1}}{\sigma_n}\frac{k_n}{k_{n+1}}\cosh(k_n d_n) \end{bmatrix} \begin{bmatrix} C_{1n} \\ C_{2n} \end{bmatrix} \tag{23}$$

which finally solves for the SE(24) of a multilayered enclosure with N layers.

$$\frac{H_o}{H_i} = \begin{bmatrix} \cosh(kd_N) \\ \sinh(kd_N) \end{bmatrix} \left(\prod_2^N \begin{bmatrix} \cosh(kd_{n-1}) & \sinh(kd_{n-1}) \\ R\sinh(kd_{n-1}) & R\cosh(kd_{n-1}) \end{bmatrix} \right) \begin{bmatrix} 1 \\ \frac{k_1}{\mu_{r1}}\frac{csa}{per} \end{bmatrix} \tag{24}$$

$$\text{with } R = \frac{\sigma_n}{k_n}\frac{k_{n-1}}{\sigma_{n-1}} \text{ and } kd_n = k_n d_n$$

It is important to note that the overall SE of a multilayered enclosure is not simply the multiplication of the SE of the separate layers, because the geometrical parameters need to be considered only once. See the example below. Equation (24) reminds of transmission line or wave propagation theory. Indeed there is a close relation which leads to the same results if the relationship to the wave impedance is considered: Z=k/σ.

For example, the SE of a copper coated enclosure is calculated, Fig.6. The quadratic base frame has a length a of 1m and a wall thickness d_1 of 1mm. Its conductivity is $\sigma_{Cu}/10000=580S/m$, whereas the coating is made of copper with a thickness d_2 of 10μm.

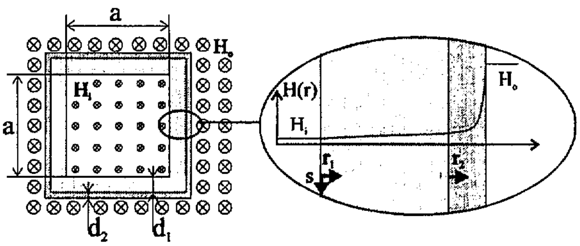


Figure 6: Magnetic field in a coated enclosure

Fig.7 shows the SE of the coated enclosure with the coating inside and outside, where no difference becomes apparent. Due to the high conductivity of the coating, even the coating alone without the basic frame has the same SE. The bottom line is the SE of the enclosure without coating. Finally, the upper line shows the added individual SEs of the basic frame and the coating, which obviously is an overestimation. This shows again that the simple addition of the individual SEs is not correct.

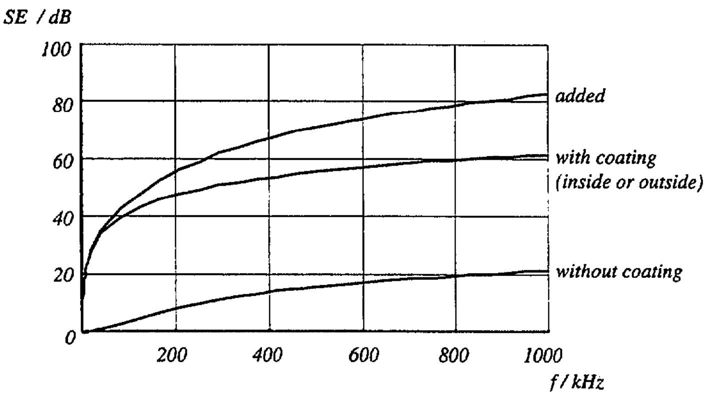


Figure 7: SE of a coated enclosure according to Figure 6

If a magnetic material is used, there is a vast difference between an inner and an outer coating, Fig.8. Due to the high permeability, the inductivity of the enclosure and as well the induced eddy currents are increased, but only if the magnetic material is inside (inner coating). Particularly a nonconducting magnetic material can considerably increase the SE for the same reasons, whereas the material alone shows only a poor SE in the frequency region considered. The magnetic coating in the example of Fig.8 has a permeability of $\mu_r=10^6$ and the same conductivity as the basic frame.

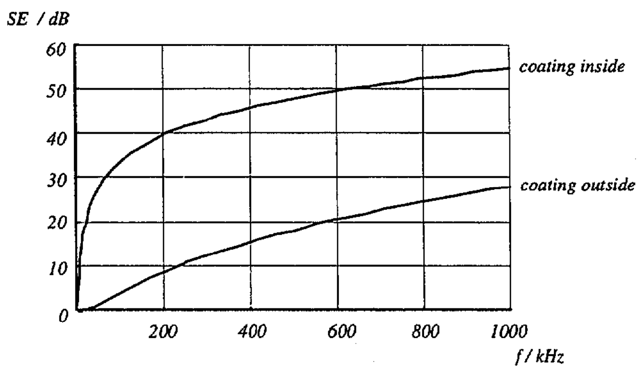


Figure 8: SE of an enclosure with a magnetic coating

Laminated and Nested Shields

Occasionally, it is assumed that the SE of enclosures can be doubled by doubling the walls or by adding shields contained within the enclosure, Fig.9. It will be demonstrated that the resulting SE is less than double. Another question of major concern is whether a thick wall can be replaced by two thin walls resulting in the saving of material and weight. Finally, the question is investigated whether an isolating gap is advantageous for the SE. In [9] a similar problem was already discussed for multishielded coaxial lines. Kaden [1] also presented analytical solutions for static magnetic fields. The derivation of the SE for frequencies below resonance frequencies is very similar to multilayer enclosures, except that there is an intermediate region with its own SE.

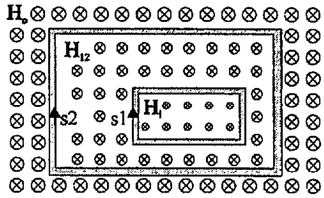


Figure 9: Nested shields

The general solution used for the magnetic field is again (4). While the boundary and continuity conditions for the magnetic field are similar to the multi-layer problems and just expanded by the fields between the shields, the continuity condition for the electric field changes.

$$
\begin{aligned}
H_{n-1}(d_{n-1}) &= H_{n-1\,n} = H_n(0) \\
E_{n-1}(d_{n-1}) &\neq E_n(0)
\end{aligned}
\qquad (25)
$$

Consequently, only the constants C_{2n} differ from the multilayer solution, the constants C_{1n} do not change. The electrical field in the region between the multiple shields is not homogeneous. Therefore, Faraday's law is applied for deriving the electric fields at the boundaries of the shield's walls.

$$
\oint E_{sn}(0)ds_n - \oint E_{sn-1}(d_{n-1})ds_{n-1} = -j\omega\mu_0 \iint H_{n-1\,n}\, dA_{n-1\,n}
$$

$$
\begin{aligned}
&\frac{k_n}{\sigma_n} C_{2,n} \cdot per_{inside,n} - \\
&\frac{k_{n-1}}{\sigma_{n-1}} \left(C_{1,n-1}\sinh(kd_{n-1}) + C_{2,n-1}\cosh(kd_{n-1})\right) \cdot per_{outside,n-1} \\
&= j\omega\mu_0 H_{n-1,n} \cdot csa_{n-1,n}
\end{aligned}
\qquad (26)
$$

The $csa_{n-1,n}$ is the area between shield n-1 and shield n. The $per_{inside,n}$ is the length of the inner contour of shield n. Because walls are so thin compared to the shield's dimension, there is almost no difference between the inner and outer perimeter, therefore, this is not indexed anymore. With (19) from the section above and (26), the constants are fully determined and the solution is again best described as a matrix equation.

$$
\frac{H_o}{H_i} = \begin{bmatrix} \cosh(kd_N) \\ \sinh(kd_N) \end{bmatrix} \left(\prod_{1}^{N-1} \begin{bmatrix} \cosh(kd_n) & \sinh(kd_n) \\ A & B \end{bmatrix} \right) \begin{bmatrix} 1 \\ \frac{k_1}{\mu_{r1}} \frac{csa_{01}}{per_1} \end{bmatrix} \qquad (27)
$$

$$
A = \frac{\sigma_{n+1}}{\sigma_n} \frac{k_n}{k_{n+1}} \frac{per_n}{per_{n+1}} \sinh(k_n d_n) + \frac{k_{n+1}}{\mu_{rn+1}} \frac{csa_{nn+1}}{per_{n+1}} \cosh(k_n d_n)
$$

$$
B = \frac{\sigma_{n+1}}{\sigma_n} \frac{k_n}{k_{n+1}} \frac{per_n}{per_{n+1}} \cosh(k_n d_n) + \frac{k_{n+1}}{\mu_{rn+1}} \frac{csa_{nn+1}}{per_{n+1}} \sinh(k_n d_n)
$$

Again, the overall SE of nested shields is *not* the addition of the SE of the single shields.

The following example investigates whether a thick wall can be replaced by two thin walls, resulting in savings of material and weight, for example, whether a 3mm wall can be replaced by two 1mm shields. Fig.10 shows the difference of the SE of a doubly shielded and a single shielded cylindrical enclosure, depending on wall thickness, respectively the outer radius. The conductivity is $\sigma_{Cu}/10000 = 580$W/m and the inner radius r_i is 0.1m. The wall thickness of the massive enclosure is just the difference between the outer and inner radius, i.e. r_o- r_i, while the double shield consists always of two cylinders of the radius r_i and r_o with an individual wall thickness of 1mm.

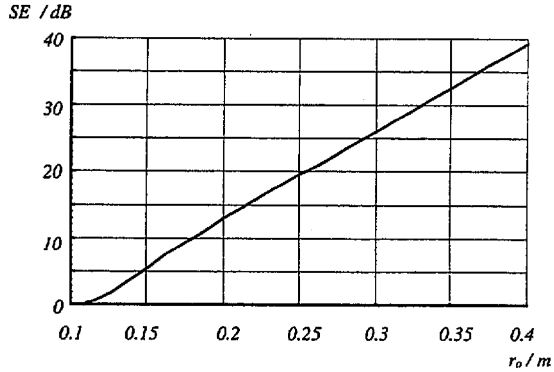
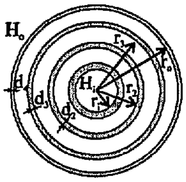


Figure 10: Difference of the SE of a doubly shielded and a single shielded cylindrical enclosure.

Obviously, the SE of the massive enclosure is always much higher. The gap generates no „insulation", therefore no material or weight savings are possible in this case.

A different question is whether, using a certain amount of material, the SE can be increased by distributing the material amongst many shields, Fig.11. In the example, the inner and outer radius are kept constant and the material is evenly distributed among N shields, Fig.11, see the correlations below. Consequently the outer shields are thinner than the inner ones.



radius: r_i=r_1=0.1m r_o=r_N=1m

constant distance: r_2-r_1=r_{n+1}-r_n=(r_o-r_i)/(N-1)

constant amount of material:

r_1d_1=r_nd_n=100mm^2/N

Figure 11: Cylindrical enclosure consisting of N shields

The material is copper (σ_{Cu}=5.8·10^{7}S/m) and the amount is determined by an overall cross sectional area of all walls of 100mm^2. Interestingly, the SE, derived from (27), increases with the number of shields, despite of the assumed constant amount of material, Fig.12. The short line to the left marks the SE of a single shielded cylindrical enclosure with a radius of r_i and of the same weight.

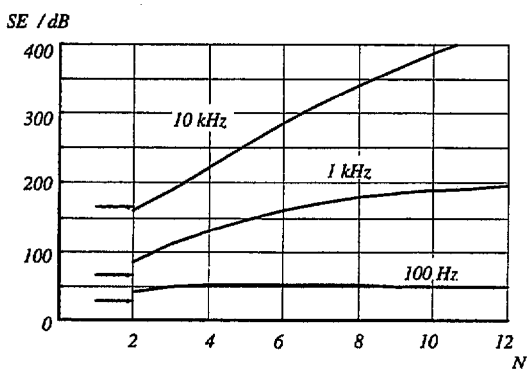


Figure 12: SE of a cylindrical enclosure consisting of N shields of copper

Employing a magnetic material with μ_r=10^6 and σ=σ_{Cu}/10000, the distribution of the material among many shields causes just the opposite effect and the SE decreases, Fig.13.

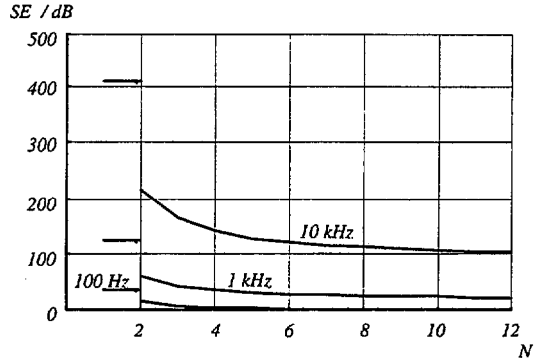


Figure 13: SE of a cylindrical enclosure consisting of N shields of magnetic material

Multi-Cavity Enclosures

If an enclosure consists of several separated cavities, Fig.14, each has its own SE (SE_1 to SE_4), which still depends on the overall geometry of the shield. For the analytical solution the enclosure's wall must be divided into sections which are common to two cavities, e.g., 1 and 2 or 1 and 4. Section 1/o is the part of the wall which exists between cavity I and the outside region. Consequently, an enclosure with N cavities has at most 3(N-1) sections.

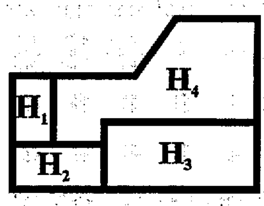


Figure 14: Multi cavity enclosure

For each section a general solution with unknown constants is needed, where $H_{x/y}$ is the magnetic field in the part of the wall between cavity x and y. The component $r_{x/y}$ is directed from cavity x to y. For simplicity, there are even two general solutions used for a section, one in each direction.

$$\begin{aligned} H_{x/y}(r_{x/y}) &= C_{1,x/y}\cosh(k_{x/y}r_{x/y}) + C_{2,x/y}\sinh(k_{x/y}r_{x/y}) \\ H_{y/x}(r_{y/x}) &= C_{1,y/x}\cosh(k_{x/y}r_{x/y}) + C_{2,y/x}\sinh(k_{y/x}r_{y/x}) \end{aligned} \tag{28}$$

The boundary conditions are

$$\begin{aligned} H_{x/y}(0) &= H_x = C_{1,x/y} \\ H_{y/x}(0) &= H_y = C_{1,y/x} \\ H_{x/y}(d_{x/y}) &= H_y = H_x\cosh(k_{x/y}d_{x/y}) + C_{2,x/y}\sinh(k_{x/y}d_{x/y}) \\ H_{y/x}(d_{y/x}) &= H_x = H_y\cosh(k_{x/y}d_{x/y}) + C_{2,y/x}\sinh(k_{y/x}d_{y/x}) \end{aligned} \tag{29}$$

Again, we need to calculate the electric field in order to obtain the SE.

$$E_{x/y}(r_{x/y}) = -\frac{1}{\sigma_{x/y}}\frac{\partial H_{x/y}}{\partial r_{x/y}} = \tag{30}$$

$$= -\frac{k_{x/y}}{\sigma_{x/y}}(C_{1,x/y}\sinh(k_{x/y}r_{x/y}) + C_{2,x/y}\cosh(k_{x/y}r_{x/y}))$$

$$E_{x/y}(0) = -\frac{k_{x/y}}{\sigma_{x/y}}C_{2,x/y}$$

Applying Faraday's law to each cavity, results in the sum of the different electric fields. The length of each section is $l_{x/y}$, the cross sectional area of a cavity x is csa_x. For example, in cavity 1 Faraday's law yields

$$\frac{k_{1/2}}{\sigma_{1/2}} C_{2,1/2} l_{1/2} + \frac{k_{1/4}}{\sigma_{1/4}} C_{2,1/4} l_{1/4} + \frac{k_{1/o}}{\sigma_{1/o}} C_{2,1/o} l_{1/o} = j\omega\mu_0 csa_1 H_1 \quad (31)$$

Similar equations can be derived for the other cavities. Substituting for the constants $C_{2,x/y}$ by (30) yields the SE of each cavity, as a system of equations though. As an example, (32) shows the result for the enclosure of Fig.14 with a constant material and wall thickness.

$$\begin{aligned} H_1 &= \frac{H_o l_{1/o} + H_2 l_{1/2} + H_4 l_{1/4}}{per_1 \cosh(kd) + k \cdot csa_1 \cdot \sinh(kd)} \\ H_2 &= \frac{H_o l_{2/o} + H_1 l_{2/1} + H_3 l_{2/3} + H_4 l_{2/4}}{per_2 \cosh(kd) + k \cdot csa_2 \cdot \sinh(kd)} \\ H_3 &= \frac{H_o l_{3/o} + H_2 l_{3/2} + H_4 l_{3/4}}{per_3 \cosh(kd) + k \cdot csa_3 \cdot \sinh(kd)} \\ H_4 &= \frac{H_o l_{4/o} + H_1 l_{4/1} + H_3 l_{4/3} + H_3 l_{4/3}}{per_4 \cosh(kd) + k \cdot csa_4 \cdot \sinh(kd)} \end{aligned} \quad (32)$$

In general, the SE of an enclosure with N cavities results in a matrix equation similar to network theory

$$\begin{bmatrix} A_1 & G_{1/2} l_{1/2} & \cdots & G_{1/N} l_{1/N} \\ G_{1/2} l_{1/2} & A_2 & \cdots & G_{2/N} l_{2/N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{1/N} l_{1/N} & G_{2/N} l_{2/N} & \cdots & A_N \end{bmatrix} \cdot \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{bmatrix} = \begin{bmatrix} G_{1/o} l_{1/o} H_o \\ G_{2/o} l_{2/o} H_o \\ \vdots \\ G_{N/o} l_{N/o} H_o \end{bmatrix} \quad (33)$$

$$\text{with} \quad A_x = j\omega\mu_0 \, csa_x + \sum_n \frac{k_{x/n}}{\sigma_{x/n}} l_{x/n} \coth(k_{x/n} d_{x/n})$$

$$\text{and} \quad G_{x/y} = \frac{-k_{x/y}}{\sigma_{x/y}} \frac{1}{\sinh(k_{x/y} d_{x/y})}$$

The right-hand vector is the source of the field, which is the field outside H_o multiplied by the length of each section. If the enclosure is made of only one material with a certain wall thickness the equation reduces to

$$\begin{bmatrix} A_1 & l_{1/2} & \cdots & l_{1/N} \\ l_{1/2} & A_2 & \cdots & l_{2/N} \\ \vdots & \vdots & \ddots & \vdots \\ l_{1/N} & l_{2/N} & \cdots & A_N \end{bmatrix} \cdot \begin{bmatrix} H_I \\ H_{II} \\ \vdots \\ H_N \end{bmatrix} = \begin{bmatrix} l_{1/o} H_o \\ l_{2/o} H_o \\ \vdots \\ l_{N/o} H_o \end{bmatrix} \quad (34)$$

$$\text{with} \quad A_x = per_x \cosh(kd) + k \cdot csa_x \sinh(kd)$$

For example let us derive the SE of a cylindrical enclosure divided into pie-type cavities, Fig.15.

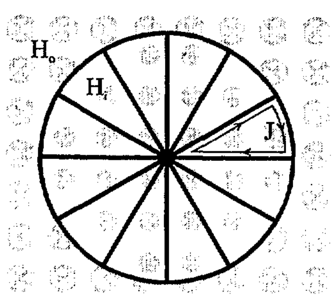


Figure 15: Cylindrical enclosure with N cavities

The result shows that the SE hardly changes and even decreases with the number of cavities, Fig.16.

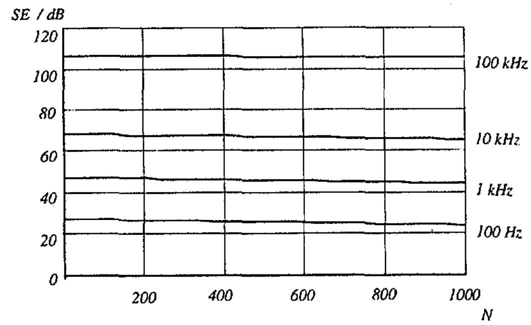


Figure 16: SE of a cylindrical copper enclosure of figure 15, depening on the number of cavities

If a magnetic material is used instead, the SE increases with the number of cavities. However, rather than the material is the reason for the enhancement of the SE, it is the increase in magnetic material inside the enclosure, resulting in a higher inductivity.

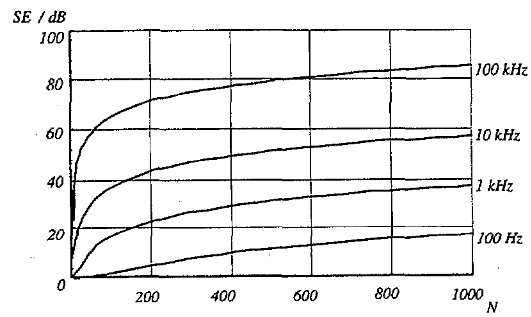


Figure 17: SE of the cylindrical enclosure of figure 15, made of magnetic material, depending on the number N of cavities

CONCLUSION

Analytical solutions for the shielding effectiveness (SE) of heterogeneous enclosures of arbitrary shape were derived. Low frequencies, where the wavelength is much greater than the dimensions of the shield, were assumed. Further, the problem was reduced to two dimensions by neglecting field distortions at edges. The influence of the shape is reflected in the ratio cross-sectional-area over the perimeter. In heterogeneous enclosures the SE is strongly determined by the weakest part. Coatings can improve the SE significantly, especially by using a magnetic inner coating and a highly conductive outer coating. The SE of nested shields is not simply the sum of the individual SEs. Finally, it was shown that the subdivision of an enclosure into several cavities does not help to increase the SE.

REFERENCES

[1] H. Kaden, Wirbelströme und Schirmung in der Nachrichtentechnik, Berlin, Heidelberg: Springer, 1959.

[2] J. J. Thomson, Recent Researches in Electricity and Magnetism, Oxford: Clarendon Press, 1893, pp. 323-328

[3] L. V. King, „Electromagnetic Shielding at Radio Frequencies", Philosophical Magazine, Journal of Science, 15, No. 97, pp. 201-223, 1933

[4] W. W. Cooley, „Low Frequency Shielding Effectiveness of Nonuniform Enclosures", IEEE Transactions on EMC, Vol.10, pp. 34-43, 1968

[5] R. W. Latham and K. S. H. Lee, "Theory of Inductive Shielding", Canadian Journal of Physics, vol.46, pp. 1745-1752, 1968

[6] P. Kistenmacher, M. Kimmel and A. Schwab, „Schirmdämpfung inhomogener Metallgehäuse beliebiger Kontur im quasistationären Magnetfeld", Electrical Engineering, vol. 78, pp. 339-344

[7] D. R. J. White and M. Mardiguian, Electromagnetic Shielding, Handbook Series on Electromagnetic Interference and Compatibility, vol.3, Interference Control Technologies, Gainesville, Virginia, 1988

[8] R. B. Schulz, V. C. Plantz, D. R. Brush, „Shielding Theory and Practice", IEEE Transactions on EMC, Vol.30, pp. 187-201, 1988

[9] Y.-P. Loh, „Shielding Theory of Coaxial Cylindrical Structures", IEEE Transactions on EMC, Vol.10, pp. 16-28, 1968

[10] P. Kistenmacher and A. Schwab, "Shielding Effectiveness of Enclosures with Arbitrary Cross Section", 1994 International Symposium on EMC, Brazil, São Paulo, pp. 281-286, 1994