Analysis of Coupling Between Transmission Lines in Arbitrary Directions

Yoshio Kami and Weikun Liu
The University of Electro-Communications
Chofu-shi, Tokyo 182, JAPAN

Abstract: A coupling between two transmission lines of finite length in arbitrary directions is studied. The transmission line generates electromagnetic fields affecting the other line so that the modified telegrapher's equation is adopted to analyze the phenomenon. One transmission line of finite length above a ground plane has risers at both line ends. Therefore the fields due to currents on the risers and the line section should be taken account. A four-port network expression is obtained from a solution to the modified telegrapher's equation. Validity of the theory is confirmed by experimental results for various models.

INTRODUCTION

A knowledge of coupling level between two transmission lines in the neighborhood is one of serious topics according as a density of interconnect wiring increases. Neighboring transmission lines, such as those in printed circuit boards, are not always in parallel but in arbitrary directions. Two transmission lines in close proximity and in parallel are referred to as coupled transmission lines, which has been studied by using transmission line theory. In this case, an electric and a magnetic couplings between them correspond, respectively, to a capacitance and an inductance matrices of the line system, so that the coupling can be estimated by using the telegrapher's equation in a matrix form or the resultant equation in terms of a four-port network matrix. When the transmission lines are of a principal propagation mode of transverse electromagnetic (TEM), it seems that the coupling between two transmission lines in the neighborhood can be approximated by considering the magnetic and the electric fields of TEM due to one line affecting the other line [1]-[3]. Though, this approaching way is somewhat defective in some practical cases [2]. When an angle between directions of two transmission lines is 90° and the transmission lines do not cross over each other, for example, it is evident to remain the electric coupling of one line to the other. Though, the other line has no component of TEM fields exciting one line. This fact shows that the reciprocity theorem does not hold in the above consideration. Moreover, there still exists the coupling in the nearby arrangement even if the TEM fields do not affect each other. For example, we can observe the coupling when the transmission lines are set closely in a straight line direction. Our subject is to obtain more meaningful approach to overcome this default.

In this paper, the line system considered here is assumed that the coupling level is less than about -15dB even for parallel lines. Two transmission lines of finite length are in arbitrary directions and have risers or vertical discontinuities at both line ends. By taking account of the fields caused by currents on both line section and risers, the coupling can be treated in a similar way to one of external fields to transmission lines, that is, the modified telegrapher's equation[4]-[6] holds in this case. A four-port network expression is resultantly obtained from a solution to the equation. Therefore, the coupling can be estimated by applying terminal conditions to the network expression. To verify the theory, comparisons between the measured and the calculated values are made by estimating parameters of scattering matrix in frequency domain; the results are in a good agreement. Moreover, the coupling responses in time domain are studied by comparing the measured and the calculated using a fast Fourier transformation.

THEORY

When a transmission line is excited by external electromagnetic fields, a current is induced on the line. This phenomenon has been studied by using a circuit concept, i.e., modified telegrapher's equation, which is expressed by a set of non-homogeneous differential equations concerned with a line voltage \( V(x) \) and a line current \( I(x) \). Forcing terms of the equation correspond to the effects due to the external fields. A solution to the modified telegrapher's equation for an isolated transmission line of length \( \ell \) is given by

\[
\begin{bmatrix}
V(\ell) \\
I(\ell)
\end{bmatrix} =
F(\ell) \begin{bmatrix}
V(0) \\
I(0)
\end{bmatrix} - \int_0^\ell F(x'-\ell) \begin{bmatrix}
V_f(x') \\
I_f(x')
\end{bmatrix} dx'
\]

(1)

where \( F(\cdot) \) is an inverse chain matrix of the transmission line concerned, and \( V_f(\cdot) \) and \( I_f(\cdot) \) are, respectively, the forcing terms denoting the magnetic field coupling and the...
electric field coupling as

\[
\begin{bmatrix}
V_f(x) \\
I_f(x)
\end{bmatrix} =
\begin{bmatrix}
-j \omega \int_0^h \mu_0 H_z^+ dy \\
j \omega C \int_0^h E_y^+ dy
\end{bmatrix}
\]

(2)

where the transmission line is assumed to be at height \( y = h \) and of a line capacitance \( C \). Terms \( H_z^+ \) and \( E_y^+ \) are a linkage magnetic field and a vertical electric field, respectively. The upper term of the right side of (2) signifies the magnetic field coupling by Faraday's law and the bottom term the electric field coupling. Those components can be rewritten in terms of a vector potential \( \mathbf{A} = (A_x, A_y, A_z) \) as

\[
\begin{bmatrix}
-\frac{1}{\omega \mu_0 \epsilon_0} \{ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \} dy \\
+ \frac{1}{\omega \mu_0 \epsilon_0} \{ \frac{\partial A_y}{\partial x} + \frac{\partial A_x}{\partial y} + \frac{\partial A_z}{\partial z} \} dy
\end{bmatrix}
\]

(3)

We consider the application of the above concept to the coupling between the neighboring transmission lines.

Figure 1 shows the model of two transmission lines in arbitrary directions. Two lossless lines \( \mathbf{f}_i (i = 1, 2) \) are installed at height \( h_i \) above the perfect ground plane. The permeability and permittivity of the space are \( \mu_0 \) and \( \epsilon_0 \), respectively, so a phase constant of the transmission lines is \( \beta = \sqrt{\mu_0 / \epsilon_0} \) because the TEM mode is assumed here. Line \( \mathbf{f}_i \) of length \( \ell_i \) are in the direction of the \( x \) and the \( X \) axes. Characteristic impedances of the transmission lines are \( Z_{0i} \) and line capacitances \( C_i \), where \( Z_{0i} \) is approximated as those of the isolated transmission lines. These two finite-length transmission lines, in practical cases, have risers at their line ends.

When there is two transmission lines in the neighbor- hood, the electromagnetic fields caused by one line affect the other line. The electromagnetic fields due to the transmission lines have been considered only as those by the TEM mode current in the line section, though, those due to the risers have been ignored almost all cases. It is pointed out that the ignored fields play an important role, for example, in printed circuit boards [7]. Therefore, we take into account of both fields due to the line section and the risers. First, we consider the fields due to line \( \mathbf{f}_2 \) affecting line \( \mathbf{f}_1 \). A vector potential at an arbitrary point \( (X, Y, Z) \) caused by the line current \( I_2(X') \) of line \( \mathbf{f}_2 \) is only of \( x \)-component:

\[
A_X = \frac{\mu_0}{4\pi} \left\{ \int_0^{r_2} \frac{I_2(X') \exp(-j\beta R_{X1})}{R_{X1}} dX' \right. \\
- \left. \int_0^{r_2} \frac{I_2(X') \exp(-j\beta R_{X2})}{R_{X2}} dX' \right\}
\]

(4)

where the second term on the bracket of the right side denotes the component due to the image current. And \( R_{Xi} (j = 1, 2) \) are

\[
R_{X1} = \sqrt{(X - X')^2 + (Y - h_2)^2 + Z^2}
\]

(5)

\[
R_{X2} = \sqrt{(X - X')^2 + (Y + h_2)^2 + Z^2}
\]

(6)

The currents on the risers generate \( Y \)-component:

\[
A_Y = \frac{\mu_0}{4\pi} \left\{ \int_{-h_2}^{h_2} \frac{I_2(0) \exp(-j\beta R_{Y1})}{R_{Y1}} dy' \\
- \int_{-h_2}^{h_2} \frac{I_2(0) \exp(-j\beta R_{Y2})}{R_{Y2}} dy' \right\}
\]

(7)

where the currents on the risers are approximated as those at \( X = 0 \) and \( X = \ell_2 \), \( I_2(\ell_2) \) and \( I_2(0) \). Terms \( R_{Yj} (j = 1, 2) \) are

\[
R_{Y1} = \sqrt{X^2 + (Y - Y')^2 + Z^2}
\]

(8)

\[
R_{Y2} = \sqrt{X^2 + (Y + Y')^2 + Z^2}
\]

(9)

Currents \( I_2(\cdot) \) on (4) and (7) can be expressed in terms of the line voltage \( V_2(0) \) and current \( I_2(0) \) at \( X = 0 \) of line \( \mathbf{f}_2 \):

\[
I_2(X') = -j \frac{V_2(0)}{Z_{02}} \sin \beta X' + I_2(0) \cos \beta X'
\]

(10)

The components expressed in the \( x-y-z \) coordinate system are obtained by using the relation as

\[
\begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
A_X \\
A_Y \\
A_Z
\end{bmatrix}
\]

(11)

By substituting the resultant expression of the vector potential due to line \( \mathbf{f}_1 \) into (3), the forcing terms, \( V_{f2} \) and \( I_{f2} \), for line \( \mathbf{f}_1 \) are obtained in a following form:

\[
\begin{bmatrix}
V_{f2}(x) \\
I_{f2}(x)
\end{bmatrix} =
\begin{bmatrix}
a_2(x) & b_2(x) \\
c_2(x) & d_2(x)
\end{bmatrix}
\begin{bmatrix}
V_2(0) \\
I_2(0)
\end{bmatrix}
\]

(12)
From (1), the resultant equation for line $\ell_1$ is

$$
\begin{bmatrix}
V_1(\ell_1) \\
I_1(\ell_1)
\end{bmatrix} =
\begin{bmatrix}
a_{11} & b_{11} & a_{12} & b_{12} \\
c_{11} & d_{11} & c_{12} & d_{12}
\end{bmatrix}
\begin{bmatrix}
V_1(0) \\
I_1(0) \\
V_2(0) \\
I_2(0)
\end{bmatrix}
$$

(13)

where parameters are

$$
a_{11} = d_{11} - \cos \beta \ell_1
$$

(14)

$$
b_{11} = -jZ_{01} \sin \beta \ell_1
$$

(15)

$$
c_{11} = -j\frac{1}{Z_{01}} \sin \beta \ell_1
$$

(16)

$$
a_{12} = \int_0^{\ell_1} \left\{ a_2(x') \cos \beta (\ell_1 - x') - jZ_{01} c_2(x') \sin \beta (\ell_1 - x') \right\} dx'
$$

(17)

$$
b_{12} = \int_0^{\ell_1} \left\{ b_2(x') \cos \beta (\ell_1 - x') - jZ_{01} d_2(x') \sin \beta (\ell_1 - x') \right\} dx'
$$

(18)

$$
c_{12} = \int_0^{\ell_1} \left\{ -j\frac{1}{Z_{01}} a_2(x') \sin \beta (\ell_1 - x') + c_2(x') \cos \beta (\ell_1 - x') \right\} dx'
$$

(19)

$$
d_{12} = \int_0^{\ell_1} \left\{ -j\frac{1}{Z_{01}} a_2(x') \sin \beta (\ell_1 - x') + c_2(x') \cos \beta (\ell_1 - x') \right\} dx'.
$$

(20)

It is noted that $a_{11}$, $b_{11}$, $c_{11}$ and $d_{11}$ correspond to those for the isolated lines because of the mentioned assumption.

By using the same procedure, the line equation for line $\ell_2$ can be obtained in a similar form. Consequently, a four-port network expression for two transmission lines in the neighborhood is obtained as

$$
\begin{bmatrix}
V_1(\ell_1) \\
V_2(\ell_2) \\
I_1(\ell_1) \\
I_2(\ell_2)
\end{bmatrix} =
\begin{bmatrix}
a_{21} & a_{22} & b_{21} & b_{22} \\
c_{21} & c_{22} & d_{21} & d_{22}
\end{bmatrix}
\begin{bmatrix}
V_1(0) \\
V_2(0) \\
I_1(0) \\
I_2(0)
\end{bmatrix}
$$

(21)

By using the above expression, the coupling or crosstalk characteristics of two transmission lines in arbitrary directions may be estimated.

**EXPERIMENT AND DISCUSSION**

To verify the theory, experiments for some models have been conducted. Two transmission lines are of 0.4mm diameter and at 4mm height from a ground plane of aluminum of $2m \times 2m$ area, so $Z_{0l} \approx 220\Omega$. Let ports 1 and 3 be the ends of line $\ell_1$, and ports 2 and 4 the ends of line $\ell_2$ as shown in Figs. 2 to 7 and 9 to 10.

![Fig. 2: $S_{21}$ for $\theta = 45^\circ$: (a) amplitude and (b) phase.](image)

![Fig. 3: $S_{41}$ for $\theta = 90^\circ$: (a) amplitude and (b) phase.](image)

**Frequency Domain**

We estimate the coupling in terms of scattering parameters of $S_{21}$ and $S_{41}$ when line $\ell_1$ is fed by a voltage source of an internal resistance 50$\Omega$ at port 1 and all other terminals of lines $\ell_1$ and $\ell_2$ are terminated with 50$\Omega$. 

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Fig. 5: $S_{41}$ for the line arrangement on a straight line: (a) amplitude and (b) phase.

Fig. 6: $S_{41}$ for the line arrangement on a right angle corner: (a) amplitude and (b) phase.

Fig. 7: $S_{41}$ for the line arrangement on an obtuse angle corner: (a) amplitude and (b) phase.

Figures 2 to 7 show comparisons in frequency domain; (a) is for amplitude and (b) for phase of $S_{21}$ and $S_{41}$. The line arrangements are shown in the figures. Figures 2 to 4 show the results for models of $l_1 = 20\text{cm}$, $l_2 = 10\text{cm}$ and $h_1 = h_2 = 4\text{mm}$ and $\theta = 45^\circ$, $90^\circ$ and $-135^\circ$. Port 4 of line $l_2$ is 5mm apart from the center of line $l_1$. In Figs. 2 and 4, there exist amplitude peaks at frequencies of about 750, 1500, 2250MHz and etc., which are resonance frequencies when $l_1$ is equal to the integer times of a half wavelength of the frequencies concerned, respectively. When $\theta = 90^\circ$ in Fig. 3, the amplitude peaks appear at frequencies of 750 and 2250MHz. Those coincide with the resonance frequencies of which a half wavelength and three halves wavelength are equal to $l_1$. In those cases, there exists so strong electric field in the vicinity of the center of line $l_1$ that the coupling peaks observed. Results of $S_{14}$, when line $l_2$ is fed at port 4 and the coupling at port 1 is measured, are quite agree with those of $S_{41}$. In the model of $\theta = 90^\circ$, in a position of line 1, the electric field of TEM mode mainly affects line $l_2$, so that the coupling magnitude becomes large at the resonance frequencies because the electric fields of TEM at those frequencies show the maximum at the center point of line $l_1$. It is noticed that the coupling levels for $\theta = 90^\circ$ are not always less than those for other models. Contrary, in a position of line $l_2$, there are no TEM fields affecting line $l_1$. Since the reciprocity theorem holds, the results show that the riser plays as an important role, i.e., the fields caused by the current on the riser affects the other line significantly.

Figures 5 to 7 show the results for models of $l_1 = 10\text{cm}$, $l_2 = 20\text{cm}$ and $h_1 = h_2 = 4\text{mm}$.
Fig. 7: $S_{21}$ for the line arrangement on a obtuse angle corner: (a) amplitude and (b) phase.

The line arrangements show in the figures; the distance between ports 1 and 2 is 2.3cm. Those measured values are in a good agreement with the calculated. Especially in the models for the arrangements of the straight line and the obtuse angle corner, Figs. 5 and 7, no coupling fields of TEM affect the transmission line at all. This fact signifies that the fields due to the current at the riser play as an important role to couple the transmission lines in the neighborhood.

**Time Domain**

Figures 9 and 10 show the results in time domain. The line arrangements are quite same as those in Figs. 2 and 3. The step voltage shown in Fig. 8 is applied to port 1; the rise time of 45 pico seconds and amplitude of 10 volts at 50Ω. The time-domain responses are measured by using a digitizing oscilloscope of a 12.4GHz bandwidth with a 50Ω input impedance. The time scale on the abscissas of the figures denotes the relative time delay because the absolute time reference can not be determined with our measurement system. The calculated values have been obtained by using a technique of a fast Fourier transform (FFT). From the discrete frequency components of the voltage source, the coupling components are obtained in frequency domain. The time-domain values are evaluated by using a technique of inverse FFT. The measured values are seemed to be in agreement with the calculated.

We consider the coupling mechanism. Time intervals between adjoining voltage peaks is about 670 pico seconds, which is corresponding to the propagation time of 20cm length line. It seems that those peaks are observed at the times when wavefronts of electromagnetic fields arrive at the line discontinuities or line ports.

In Fig. 9, the first peak A of $v_1(t)$ is less than peak A' of $v_{21}$. This is the reason why the induced current due to the magnetic field flows in the direction from port 4 to port 2 of line #2 and one due to the electric field flows in both directions toward ports 4 and 2 when the positive voltage travels from port 1 to port 3 of line #1, therefore, the resultant current at port 4 becomes the difference of them and
one at port 2 becomes the summation. The rise time of peak A' of \( v_{21} \) is longer than one of \( v_{41} \). This is interpreted by the fact that port 2, at first, picks up directly the fields generated by the riser current at port 1 and then the component mentioned above arrives. The same phenomenon appears at the first peak of \( v_{21} \) in Fig. 10 (c). A negative peak is measured right after the first positive peak; this is caused by a negative voltage component which is a part of the received one at port 4 traveling toward port 2.

The second peak B of \( v_{41} \) is mainly caused by the following reason. When the negative reflection wave travels back to port 1, port 4 is at such situation as port 2 when the positive incident wave travels along line \( \ell \). And also peak B' of \( v_{21} \) is as similar as peak A of \( v_{41} \) but sign. The peaks, of course, contain components of reflected waves at ports 2 and 4. Thus, the time responses are observed as those containing the multi-reflection components from all terminals.

The measured responses in Fig. 9 converge at zero according as time increases, though the calculated do not because of lossless assumption. Those in Fig. 10 converge soon at zero because the positive and negative values appear alternately.

**Conclusion**

We have studied the coupling between two transmission lines of finite length in arbitrary directions or in the neighborhood. The finite-length transmission lines in many cases have the risers at both ends. The electromagnetic fields due to the currents at the line section and the risers of one line affect the other line. Applying this consideration to the modified telegrapher’s equation, we have obtained the four port network expression for the voltages and the currents at terminals of both transmission lines. The couplings for some line-arrangement modes have been studied by using the network expression in frequency and time domains: comparisons between the calculated and the measured values were in a good agreement. From the results, it has been made clear that the riser plays an important role when estimating the coupling between two transmission lines in the neighborhood.

**References**


