Stability Analysis of a High Temperature Reactor Cooled by Supercritical Light Water

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Thermal-hydraulic and thermal-nuclear coupled stabilities are analyzed for a high temperature reactor cooled by supercritical light water. Frequency domain analysis is carried out using linearized perturbation equations. The result shows that the stability is kept by setting a proper orifice pressure loss coefficient. The required coefficient is not large. Sensitivities of principal parameters are investigated. There is no specific parameter which has a remarkably high sensitivity.

KEYWORDS: SCR, stability, Thermal-hydraulic stability, Thermal-nuclear coupled stability, Frequency domain analysis, Decay ratio

I. Introduction

High temperature and high performance light water cooled reactors, which are also called supercritical water cooled reactors (SCRs), have been studied by Oka et al1-4. The core is cooled by supercritical water, where boiling and boiling transition do not take place. The coolant is supplied by feedwater pumps, heated to about 500˚C in the core and directly led to the turbine. This is once-through direct cycle which is much simpler than the current systems of light water cooled reactors (LWRs).

Although there is no phase change in the core, the coolant density largely decreases. Thus, flow instabilities should be considered in the reactor design like boiling water reactors (BWRs).

In the present study, linear analyses of thermal-hydraulic and thermal-nuclear coupled instabilities are carried out for SCRs. Sensitivities of typical design parameters are also investigated and the design area is discussed.

II. Linear Stability Analysis

1. Reactor Model

The SCR reactor model is shown in Fig.1. The model consists of four parts: thermal-hydraulics, nuclear physics, fuel rod heat transfer, and ex-core system.

The core thermal-hydraulics is represented by a single channel. A mass conservation equation, an energy conservation equation and a momentum conservation equation are used as shown below. The fuel channel is axially divided for discretization.

\[
\frac{dP}{dz} + \frac{dG}{dt} + \frac{d}{dz} (\rho u^2) = \rho g \cos \theta + \frac{2f}{D_h} \frac{G^2}{\rho}
\]  

The Blausius friction factor is used.

\[
f = 0.0791 \times Re^{-0.25} \quad (2100 < Re < 10^5)
\]

Point kinetics equations with six delayed neutron groups are used for the nuclear physics. We consider the Doppler coefficient and the coolant density coefficient of reactivity. The reactivity is obtained as the average in the axial direction weighted by the square of the power distribution.

\[
\frac{dn(t)}{dt} = \frac{\Delta \rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t)
\]

\[
\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t)
\]

Fuel pellet
Coolant
Cladding tube
Gap
Orifice

Feedwater pipe
Feedwater pump
Turbine control valve

Fig.1 Reactor model
\[ \delta \rho(t) = \frac{d \Delta \rho}{dt} \delta \rho(t) + \frac{d \Delta \rho}{d \rho} \delta \rho(t) \] (7)

The fuel rod is also divided in the axial direction. Heat conduction in the pellet, gap conductance, heat conduction in the cladding and heat transfer to the coolant channel are considered. The Dittus-Boelter correlation is used for the heat transfer.

\[ \rho f (r) \frac{dT_f}{dr} = 1 \frac{d}{d \rho} \left[ \frac{d T_f}{d \rho} \right] + q^* \] (8)

\[ k_f \frac{d T_f}{dr} = q^* (r, t) = h_c (T_c - T_i) \] (9)

\[ Nu = 0.023 Re^{0.8} Pr^{0.4} \] (10)

An orifice at the channel entrance, a feedwater pump, an exit valve and a feedwater pipe are considered as the model of the ex-core system. The orifice model is

\[ \Delta P = \zeta \frac{d u^2}{2} \] (11)

The characteristics are linearized for the exit valve and the feedwater pump model.

\[ \Delta P = \zeta \frac{d u^2}{2} \] (12)

The friction pressure loss in the feedwater pipe is calculated by

\[ -\frac{d P}{d z} = \rho \frac{d u}{d t} + \rho \frac{d u}{d z} u^2 + \frac{2 f}{D} \rho u^2 \] (13)

The ex-core model is used for thermal-nuclear coupled stability.

2. Frequency Domain Analysis of Linearized Equations

We linearize and apply Laplace transform to all equations for perturbations. An upstream finite difference scheme is used for the axial discretization. For example, Eq.(1) is transformed to

\[ \frac{\partial \rho_i}{\partial t} + \frac{\rho_i u_i - \rho_{i-1} u_{i-1}}{\Delta z} = 0 \] (14)

The Laplace transform is applied to Eq.(14) and the following equation is obtained.

\[ \left[ s + \frac{u_i}{\Delta z} \right] \delta \rho_i + \left[ \frac{\rho_i}{\Delta z} \right] \delta t_i + \left[ -\frac{u_{i-1}}{\Delta z} \right] \delta \rho_{i-1} + \left[ -\frac{\rho_{i-1}}{\Delta z} \right] \delta t_{i-1} = 0 \] (15)

The other equations are transformed by the same way.

Since the coolant channel is axially discretized using the upstream scheme, a set of the transformed equations are written as,

\[ \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{u_{i-1}}{\Delta z} & -\frac{\rho_{i-1}}{\Delta z} & 0 & 0 \\ \frac{u_{i-1}}{\Delta z} & \frac{\rho_{i-1}}{\Delta z} & -1 & -\frac{\rho_{i-1} u_{i-1}}{\Delta z} \\ \frac{h_{i-1} u_{i-1}}{\Delta z} & \frac{h_{i-1} \rho_{i-1}}{\Delta z} & 0 & \frac{\rho_{i-1} u_{i-1}}{\Delta z} \end{bmatrix} \begin{bmatrix} \delta \rho_{i-1} \\ \delta t_{i-1} \\ \delta \rho_{i-1} \\ \delta t_{i-1} \end{bmatrix} + \begin{bmatrix} s + \frac{u_i}{\Delta z} & \frac{\rho_i}{\Delta z} & 0 \\ 0 & s + \frac{u_{i-1}}{\Delta z} & \frac{\rho_{i-1}}{\Delta z} \\ 0 & 0 & s + \frac{u_{i-1}}{\Delta z} \end{bmatrix} \begin{bmatrix} \delta \rho_i \\ \delta t_i \\ \delta \rho_{i-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \delta \rho_i \\ \delta t_i \\ \delta \rho_{i-1} \end{bmatrix} \] (16)

\[ \Gamma_A = -\frac{2 u_{i-1} \rho_{i-1}}{\Delta z} + \left( \frac{4 f}{D_h} \right) u_{i-1} \rho_i \] (17)

\[ \Gamma_B = -u_i s + \frac{u_i^2}{\Delta z} + g \cos \theta + \left( \frac{2 f}{D_h} \right) u_{i-1}^2 \] (18)

The relation between the core inlet and outlet is obtained by combining Eq.(16)

A feedback system involving a forward transfer function \( G \) and a feedback transfer function \( H \) is considered. The total transfer function of the feedback system is

\[ \frac{G(s)}{1 + G(s)H(s)} \] (19)

Functions \( G \) and \( H \) are shown in Figs.2 and 3 for thermal-hydraulic and thermal-nuclear coupled instabilities, respectively.

The characteristic equation

\[ 1 + G(s)H(s) = 0 \] (20)

gives the poles of the closed-loop transfer function. Among the poles, the nearest pole to the complex axis represents the response of the system, which is used to calculate the decay ratio, \( \chi / \chi_0 \) as shown in Fig.4. The decay ratio is defined as the proportion by which the amplitude decays in one cycle for a given step disturbance. When the mesh size is increased in the axial direction, the frequency response approaches a converged value. Therefore, the decay ratio is calculated as extrapolation at the mesh size being zero. The method of least squares are used for the extrapolation. Figure 5 shows a typical result of dependences of the mesh
size with respect to the pressure loss coefficient of the inlet orifice, $\zeta$.

### III. Stability Analysis

The stability criteria of SCRs are applied the same as BWRs. The criteria are summarized in Table 1.

A high-temperature supercritical light water reactor (SCLWR-H) is analyzed. The characteristics of SCLWR-H are summarized in Table 2.

Partial-power as well as full-power operations are analyzed. The mass velocity decreases in proportion to the power decrease. The inlet and the outlet temperatures are kept constant. This is because the instability likely to occur at partial-power operations during start-up. In the past studies, two types of start-up procedures were proposed for SCRs: constant-pressure and variable-pressure start-up. In the present study, the code is developed for supercritical pressure, so that the result can be applied to the constant-pressure start-up procedure. We need another code to develop at subcritical pressure for the variable-pressure start-up procedure.

#### Table 1  Criteria of stability

<table>
<thead>
<tr>
<th></th>
<th>All operation</th>
<th>Normal operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal-hydraulic</td>
<td>$x_2 / x_0 &lt; 1.0$</td>
<td>$x_2 / x_0 &lt; 0.50$</td>
</tr>
<tr>
<td>stability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal-nuclear</td>
<td>$x_2 / x_0 &lt; 1.0$</td>
<td>$x_2 / x_0 &lt; 0.25$</td>
</tr>
<tr>
<td>coupled stability</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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**Fig. 2** Diagram of thermal-hydraulic stability

**Fig. 3** Diagram of thermal-nuclear coupled stability

**Fig. 4** Decay Ratio

**Fig. 5** Convergence of decay ratio with mesh size
1. Thermal-Hydraulic Stability

Maximum power and average power channels are analyzed for the thermal-hydraulic stability. The frequency responses of the full power of the average channel at orifice loss coefficient $\zeta=10$ are shown in Fig.6. The axial power distribution is assumed to be a cosine function. We can see a peak of the gain at $\omega=3$ which corresponds to the time when the coolant flows through the core. This peak is related to the possible instability.

**Table 2** Characteristics of SCLWR-H

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal/Electric Power</td>
<td>[MW]</td>
<td>3568/1570</td>
<td></td>
</tr>
<tr>
<td>Core height</td>
<td>[m]</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>[MPa]</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>Fuel/Cladding material</td>
<td>UO$_2$/Inconel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel rod diameter/</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cladding thickness/Pitch</td>
<td>[mm]</td>
<td>8.0/0.40/9.5</td>
<td></td>
</tr>
<tr>
<td>Inlet/outlet coolant Temp</td>
<td>[°C]</td>
<td>280/508</td>
<td></td>
</tr>
<tr>
<td>Doppler coefficient</td>
<td>[(pcm/K)]</td>
<td>-2.50</td>
<td></td>
</tr>
<tr>
<td>Coolant density coefficient</td>
<td>[(dk/k)/(g/cc)]</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>Maximum/Average linear power</td>
<td>[W/cm]</td>
<td>390/246.5</td>
<td></td>
</tr>
<tr>
<td>Mass velocity</td>
<td>[kg/m²s]</td>
<td>1310</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7 shows the relation between the power and the decay ratio. As the power decreases, the core tends to be unstable till 30% of the full power. However, the stability is improved when the power is below 30%. Figure 8 shows the relation between the orifice pressure loss coefficient and the maximum decay ratio in the power range from 10 to 100%. The stability is improved as the orifice pressure loss coefficient $\zeta$ increases. If $\zeta>6.8$ (the pressure loss is 0.17bar), the thermal-hydraulic stability satisfies the criteria of the normal operation. This loss coefficient is not large.

In SCLWR-H, we need care the thermal-hydraulic instability. However, the instability can be avoided when the orifice pressure loss coefficient is properly designed. The value of the coefficient is not large.

**Figure 6** Frequency response of thermal-hydraulic stability

**Figure 7** Relation between power and decay ratio of thermal-hydraulic stability

**Figure 8** Relation between orifice pressure loss coefficient and maximum decay ratio of thermal-hydraulic stability
2. Thermal-Nuclear Coupled Stability
The average power channel is analyzed in the thermal-nuclear coupled stability. The response is shown in Fig.9. We can see a peak of gain at $\omega=3$ when the power is 100%. This frequency is the same as that of the thermal-hydraulic stability. As the power decreases, the peak is larger and moves to a lower frequency. This is due to the decrease of the flow velocity. When the flow velocity decreases, the time which is necessary to flow through the core is longer.

Figure 10 shows the relation between the power and the decay ratio. The calculation result shows that the stability is worse as the power decreases though the maximum value is lower than the criteria. The decay ratio is 0.028 at the normal operation, which satisfies the criterion. The maximum decay ratio is 0.178 at the partial-power operations, which satisfies the criterion of the decay ratio for all operations.

IV. Sensitivity
Sensitivity of several principal parameters is studied. They are the core height, the linear power, the mass velocity, the coolant density coefficient and the axial power distribution. The reference core is SCLWR-H at 100% power operation. The characteristics are shown in Table 3.

1. Core Height
In the once-through direct cycle of SCRs, the coolant flow rate in the feedwater line, the core and the main steam line is the same. This requires that the flow velocity in the core must be enhanced by employing a tight lattice fuel assembly or by increasing the core height. A longer core height makes the coolant velocity faster. Thus, the effect of the core height on the stabilities is investigated.

The core height is changed by keeping the linear power and the inlet and outlet temperatures. Figures 11 and 12 shows the results of the thermal-hydraulic and thermal-nuclear coupled stabilities, respectively. When the core height is increased, the thermal-hydraulic stability is a little better and the thermal-nuclear coupled stability is a little worse. The sensitivity is low and we do not need consider the effect of the core height on the stabilities.

2. Linear Power
Sensitivity of the linear power to the thermal-hydraulic and thermal-nuclear coupled stabilities is investigated. The mass velocity and the core inlet temperature are kept constant. The results are shown in Figs.13 and 14. When the linear power increases, the decay ratio of the thermal-hydraulic stability increases but that of the thermal-nuclear stability is almost constant. For the thermal-hydraulic stability, the decay ratio exceeds 0.5 at 114% power. It exceeds 1.0 at 136% power.

The linear power cannot be enhanced in the steady-state operation as long as UO₂ or MOX are used as the fuel. The linear power increases in transient conditions. In the past studies of the safety analysis, the reactor scram signal is triggered.

Table 3 Reference core characteristics of SCLWR-H for sensitivity analysis

<table>
<thead>
<tr>
<th>Core height [m]</th>
<th>Mass velocity [kg/m²s]</th>
<th>Linear power [W/cm]</th>
<th>Coolant density coefficient [(dk)/g/cc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>1310</td>
<td>246.5</td>
<td>0.40</td>
</tr>
</tbody>
</table>
released when the power is over 110%. Thus, the thermal-hydraulic instability will not be realized.

3. Mass Velocity

The core outlet temperature is about 500°C in the SCLWR-H. A higher core outlet temperature is achieved by reducing the feedwater flow rate. This reduces the size of the BOP (balance of plant) as well as enhances thermal efficiency. Thus, sensitivity of the mass velocity is investigated. The linear power and the core inlet temperature are kept constant. The core outlet temperature changes with the mass velocity.

Figures 15 and 16 show the relations between the mass velocity and the decay ratio in the thermal-hydraulic and thermal-nuclear coupled instabilities. The decay ratio increases as the mass velocity decreases. The thermal-hydraulic stability is severer. The decay ratio is 0.5 at 88% mass velocity and it is 1.0 at 73% mass velocity. We need to increase the orifice pressure loss coefficient when the mass velocity is reduced. The present coefficient of \( \zeta = 6.8 \) is not large. In transients, the reactor scram signal is released at 90% flow rate, which is in the stable region. Although the sensitivity of the mass velocity is high, it is not difficult to keep the reactor stability in the design.

4. Coolant Density Coefficient

Sensitivity of the coolant density coefficient to the thermal-nuclear coupled stability is examined. The liner power, the mass velocity and the inlet and outlet temperatures are not changed. Figure 17 shows the result. The sensitivity is high at the reference point, but the relation is nonlinear. The criterion for the normal operation is satisfied till the coolant density coefficient increases to 200~225 times as large as the reference value of \( 0.40 (\text{g/cc}) \). This is not realistic.

5. Axial Power Distribution

In the experience of BWR, stabilities are worse when the axial power distribution has a peak at the bottom part. A test
calculation is carried out using two distributions of a symmetric cosine and a bottom peak (Fig. 18). The bottom peak is located at the 1/4 height of the core. The total power is the same.

Figures 19 and 20 show the decay ratio of the thermal-hydraulic and the thermal-nuclear coupled instabilities. When the power distribution has a peak in the bottom, the stabilities are worse. The sensitivity is not high. In the detailed design of SCRs, we need to evaluate the stabilities using the severest axial power distribution.

V. Conclusions
A stability analysis code for supercritical water cooled reactors (SCRs) is developed for thermal-hydraulic and thermal-nuclear coupled stabilities.

The stability analysis is carried out for a high-temperature supercritical light water reactor (SCLWR-H). The maximum decay ratio appears in partial power operations. We need to design the orifice pressure loss coefficient at the inlet of the fuel assemblies. The coefficient required for SCLWR-H is not large.

Sensitivities of five parameters are investigated. They are the core height, the linear power, the mass velocity, the coolant density coefficient and the axial power distributions. There is no specific parameter which has a remarkably high sensitivity.

From this study, we can conclude that stabilities are need to consider in the SCR design. It is not difficult to keep the stability involving partial power operations by the design of the orifice pressure loss coefficient at the inlet of the fuel assemblies.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>area</td>
</tr>
<tr>
<td>C_i</td>
<td>number density of delayed neutron precursor of group i</td>
</tr>
<tr>
<td>C_p</td>
<td>specific heat</td>
</tr>
<tr>
<td>D_h</td>
<td>hydraulic diameter</td>
</tr>
<tr>
<td>f</td>
<td>friction factor</td>
</tr>
<tr>
<td>G</td>
<td>flow rate, forward transfer function</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity</td>
</tr>
<tr>
<td>H</td>
<td>feedback transfer function</td>
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<tr>
<td>h</td>
<td>enthalpy</td>
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References


