An Integrated Approach in the Structural Design of Fast Breeder Reactors

Part 2: A Simplified Procedure for Margin Exchange Evaluation

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For the commercialization of fast breeder reactors, “System Based Code”, a completely new scheme of a code on structural integrity, is being developed. One of the distinguished features of the System Based Code is that it is able to determine a reasonable total margin on a structural system, by allowing the exchanges of margins between various technical items. Detailed estimation of failure probability of a given combination of technical items and its comparison with a target value is one way to achieve this. However, simpler and easier methods that allow margin exchange without detailed calculation of failure probability are desirable in design. Therefore, the authors newly developed a “Vector Method”.

KEYWORDS: Failure probability, Fatigue, Margin exchange

I. Introduction

One of the promising methods to develop the System Based Code (SBC) proposed by Asada et al. [1,2] is the use of probabilistic methods in structural integrity assessment. Detailed estimation of failure probability of a given combination of technical items and its comparison with a target value is one way to achieve this [3-5]. However, simpler and easier methods that allow margin exchange without detailed calculation of failure probability are desirable for practical use in design.

One of the difficulties of estimation of failure probability is that the effect of a change of a particular variable on failure probability depends on the values of other variables, which is the basic characteristic of a joint probability density function. In order to avoid this difficulty, the authors developed a new “Vector Method”. This method first calculates failure probabilities for various combinations of design variables, and then, variables are transformed so that failure probability is expressed as a linear summation of each variable. This method allows easy and practical evaluation of margin exchange and gives insight as to the possibility of an effective margin exchange.

II. Concept of “Vector Method”

1. Premise

A simplified method for margin exchange evaluation that can be incorporated to a System Based is to fulfill the following requirements:

1) It has to have enough accuracy in terms of failure probability assessment to apply to design evaluation.
2) It can easily perform margin exchange evaluation for a number of combinations of design variables.
3) It can give insight to the judgment of the possibility of margin exchange and relative significance of each design variable.

From this viewpoint, the authors developed a new method, which is called a “Vector Method”.

2. Equi-failure probability surface

When a failure probability of a component \( P_f \) is a function of design parameters \( x_1, x_2, \ldots, x_n \), then \( P_f \) can be expressed by Equation (1):

\[
P_f = G(x_1, x_2, \ldots, x_n)
\]

Equation (1) expresses an equi-failure surface in an \( n \)-dimensional space (We call this space \( X \) hereafter). Each point in the space \( X \) represents a combination of \( n \) design variables. Between any two points on an equi-failure surface, margin exchange is possible, which means the two points correspond to the same value of failure probability despite the difference in the values of design parameters.

If “main variables” \( x_1, x_2, \ldots, x_m \) are determined by “sub variables” \( x_{11}, x_{12}, \ldots, x_{m_n} \), then, Equation (1) becomes Equation (2):

\[
P_f = G(x_1, x_{12}, \ldots, x_{m_n}, x_2, x_{22}, \ldots, x_{m_n}, \ldots, x_{n}, x_{n2}, \ldots, x_{mn})
\]

3. Equi-failure probability plane

When an equi-failure probability surface is non-linear, margin exchange using Equation (1) may not be very easy and practical. However, if it is a plane, margin exchange will be easy and insight as to the relative significance of each variable in terms of the effect on failure probability will also be obtained. Therefore, we consider the possibility of a projection \( g \) which projects an equi-failure probability surface in space \( X \) to an equi-failure probability plane in space \( X' \):

\[
g : X \rightarrow X'
\]
By the projection \( g \), a surface \( G \) is projected to a plane \( H \). Equations corresponding to (2) and (3) are as follows:

\[
H = (A|X) = (a_1, a_2, \ldots, a_x) \begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_x' \end{pmatrix}
= (a_1, a_2, \ldots, a_x) \begin{pmatrix} (A_1|X_1) \\ (A_2|X_2) \\ \vdots \\ (A_x|X_x) \end{pmatrix}
\]

Where,

\[
x'_i = (A_i|X_i) = (a_{i1}, a_{i2}, \ldots, a_{im(i)}, a_{im(i)+1}) \begin{pmatrix} x_{i1}' \\ x_{i2}' \\ \vdots \\ x_{im(i)}' \\ 1 \end{pmatrix}
\]

where, \( A \) is a coefficient vector and \( X \) is a design variable vector.

4. Quality Assurance plane

An equi-failure probability plane (Equation (4)) is easier to use compared to an equi-failure probability surface (Equation (1)). However, because design variables are expressed in terms of physical quantities in Equation (4), very large numbers or small numbers have to be dealt with, which may be sometimes troublesome in margin exchange evaluations. Therefore, the applicability of “Quality assurance index” proposed by Asada et al. [2] is examined.

The quality assurance index is expressed by Equation (5).

\[
F = C(I)Q(I, J_i) + C(2)Q(I, J_2) + \cdots + C(K)Q(I, J_k)
\]  

(5)

Where,

- \( Q(I, J_i) \): Quality assurance index of the \( i \)th option in the \( I \)th partial code
- \( C(I) \): Influence coefficient for the \( I \)th partial code
- \( F \): Total Quality assurance index

Equation (5) can be rewritten in a vector form:

\[
F = (C|Q)
\]  

(6)

\( Q \) is a quality assurance index vector and \( C \) is an influence coefficient vector:

\[
Q = q_1e_1 + q_2e_2 + \cdots + q_ne_n = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} + \cdots + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix}
\]

\[
C = c_1e_1 + c_2e_2 + \cdots + c_ne_n = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}
\]  

(7)

Equation (6) represents plane \( F \) in an \( n \) dimensional space. The influence coefficient vector \( C \) is a normal vector of the plane \( F \). The total quality assurance index \( F \) corresponds to the distance between the origin and the surface.

Equation (4) and (7) correspond very well. Therefore, we consider a projection \( h \) from space \( X' \) to a space where the quality assurance plane exists (We call this space \( Q \) hereafter):

\[
h : X' \rightarrow Q
\]  

(8)

By the projection \( h \), an equi-failuer probability surface \( H \) is projected to a quality assurance plane \( F \).

\[
F = (C|Q) = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix} = \begin{pmatrix} (c_1q_{i1}) \\ (c_2q_{i2}) \\ \vdots \\ (c_nq_{in}) \end{pmatrix}
\]  

(9)

Where,

\[
q_i = (C_i|Q_i) = \begin{pmatrix} c_{i1} \\ c_{i2} \\ \vdots \\ c_{in} \end{pmatrix}
\]

\( q_i \) is determined such that when design parameter moves from minimum to maximum, \( q_i \) moves from 0 to 5, 0 corresponding to minimum quality and 5 corresponding to the maximum quality for example.

5. Margin exchange evaluation

A designer can perform margin exchange evaluation using Equation (9). First, a total quality assurance index \( F_0 \) has to be chosen, according to the reliability required for the component being designed. The relationship between reliability (failure probability) and the total quality assurance index \( F \) is calculated using Eq. (9). If \( F \) meets \( F_0 \) for a particular combination of design options, that design holds. If there are more than one combination that satisfy \( F_0 \), then margin exchange holds among those
combinations. A designer can choose one that is most preferable in terms of cost, regulatory requirements, etc.

III. Example of determination of quality assurance index
In this section, a worked example of margin exchange based on the Vector Method is shown.

1. Problem description
A welded joint in a structure subjected to low cycle fatigue to which in-service inspection (ISI) was applied, was considered. Failure was defined as a point when crack depth reached 3/4 of $t$. Crack propagation rate in a structure, accuracy of ISI (probability of detection of cracks), and the frequency of ISI were selected as main design variables. The crack propagation rate here is not material property but the one in structure, which is determined by material property, loading, as well as configuration of the structure. Therefore, crack propagation rate was considered to be a function of two sub variables, that is, load level and material property. A fish bone diagram for this problem is shown in Fig. 1.

Other details of this example problem is as follows:
1) Configuration: Cylinder (Outer diameter: 508mm, thickness: 12.7mm)
2) Maximum depth of initial defect: 9.5mm
3) Number of cycles at which failure probability is calculated: 1000cycles

2. Flow of failure probability calculation
Crack propagation rate in the structure is assumed as Eq. (10):

$$\frac{da}{dn} = g(a) = ca^n$$  \hspace{1cm} (10)

Where, $a$ is crack depth, $c$ and $m$ are constants corresponding to a particular combination of load level and material property.

Crack depth at the $n$th cycle $a(n)$ is given by Eq. (11):

$$a(n) = \left\{ a_0^{1-m} + c(1-m)n \right\}^{\frac{1}{m}}$$  \hspace{1cm} (11)

Where, $a_0$ is initial depth.

A probability density function of initial cracks is assumed as Eq. (12):

$$f(a_0) = \frac{3(a_{\text{max}} - a_0)^2}{a_{\text{max}}^3}$$  \hspace{1cm} (12)

Cumulated failure probability at the $n$th cycle is the ratio of cracks whose depth is greater than the critical crack depth. Therefore, it is given by equation (13):

$$P_f(n) = \int_{a_{\text{cr}}}^{a_{\text{max}}} f(a_0)da_0$$

$$= \int_{a_{\text{cr}}}^{a_{\text{max}}} \frac{3(a_{\text{max}} - a_0)^2}{a_{\text{max}}^3} da_0$$

$$= \frac{(a_{\text{max}} - a_{\text{cr}})^3}{a_{\text{max}}^3}$$

$$= \frac{1}{a_{\text{max}}^3} \left\{ a_{\text{cr}}^{1-m} - c(1-m)n \right\}^{\frac{1}{m}}$$  \hspace{1cm} (13)

Failure probability at the $n$th cycle is obtained by differentiating Eq. (13):

$$\frac{dP_f(n)}{dn} = \frac{3c(m-1)}{a_{\text{max}}^2} \left\{ a_{\text{cr}}^{1-m} - c(1-m)n \right\}^{\frac{1}{m}}$$

$$\times \left\{ \frac{1}{m-1} \left\{ a_{\text{cr}}^{1-m} - c(1-m)n \right\}^{\frac{1}{m}} \right\}$$  \hspace{1cm} (14)

When ISI accuracy (probability of detection of cracks) is assumed a constant independent of crack size, cumulated failure probability at the $n$th cycle considering the effect of ISI is given by Eq. (15):

$$P_{\text{cum}}(n) = \sum_{i=0}^n \left(1 - POD\right)^i (P_f(n_i) - P_f(1))$$  \hspace{1cm} (15)

3. Simplified method of calculating failure probability
In general, Eq. (15), which represents cumulated failure probability when ISI is periodically applied, can be very complex, and usually numerical method such as Monte-Carlo simulation is employed. However, it is very beneficial if failure probability can be calculated by simpler method. Therefore, a simplified method that calculates failure probability algebraically without using numerical methods is examined in this paper.

When $m$ is assumed to be 0.5, Eq. (15) can be written as follows:

$$P_{\text{cum}}(n) = A_{1}\left\{ \sum_{i=0}^n b_i (1 - POD)^i + \sum_{s=1}^k c_i q_i (1 - POD)^s \right\}$$  \hspace{1cm} (16)

Where,

$$A_{1} = A_1(a_{\text{max}})$$

$$b_i = b_i(a_{\text{max}}, a_{\text{cr}}, c, m, n_0)$$

$$c_i = c_i(a_{\text{max}}, a_{\text{cr}}, c, m, n_0, n)$$

$$n = \ln n_0 + p$$
4. Equi-failure probability surface in X space

In this section, an equi-failure surface was calculated using Eq. (16), three main variables varying within the following range:

c: 0.0005 – 0.01
POD: 0.1 – 0.9
n: 2 – 1000 cycles

5 equi-failure surfaces were calculated at n=1000 cycles for the failure probability of 1x10^-3, 1x10^-4, 1x10^-5, 1x10^-6, and 1x10^-7. An equi-failure probability for 1x10^-3 is shown in Fig. 2 as an example.

This surface can be approximated by Eq. (17):

\[
q_1 = 1 + 4 \times \frac{\log \left( \frac{POD}{POD_0} \right) - \log \left( \frac{POD}{POD_{0,\min}} \right)}{\log \left( \frac{POD}{POD_{0,\max}} \right) - \log \left( \frac{POD}{POD_{0,\min}} \right)}
\]

An example of quality assurance plane for 1x10^-5 is shown in Fig. 4.

5. Equi-failure probability surface in space X'

Projection from space X to space X' is examined. Logarithm of Eq. (17) yields Eq. (18):

\[
q_1 = 5 \times \frac{\log \left( \frac{1}{n_0} \right) - \log \left( \frac{1}{n_0_{\min}} \right)}{\log \left( \frac{1}{n_0_{\max}} \right) - \log \left( \frac{1}{n_0_{\min}} \right)}
\]

An equi-failure surface X' for 1x10^-3 is shown in Fig. 3. In this case, b_2 and b_3 are far smaller than b_1 and Eq. (18) can be rewritten as Eq. (19).

\[
q_1 = 1 \times \frac{\log \left( \frac{POD}{POD_0} \right) - \log \left( \frac{POD}{POD_{0,\min}} \right)}{\log \left( \frac{POD}{POD_{0,\max}} \right) - \log \left( \frac{POD}{POD_{0,\min}} \right)}
\]

Equation (19) represents a plane in terms of slog x_i.

6. Quality assurance plane in space Q

To project a plane X' represented by Eq.(19) to a quality assurance plane, linear transformation is applied to each variable. In this paper, quality assurance indices were determined so that an index moves from 0 or 1 to 5 when a design variable moves from minimum to maximum:

\[
q_2 = 1 + 4 \times \frac{\log \left( \frac{POD}{POD_0} \right) - \log \left( \frac{POD}{POD_{0,\min}} \right)}{\log \left( \frac{POD}{POD_{0,\max}} \right) - \log \left( \frac{POD}{POD_{0,\min}} \right)}
\]

\[
q_3 = 5 \times \frac{\log \left( \frac{1}{n_0} \right) - \log \left( \frac{1}{n_0_{\min}} \right)}{\log \left( \frac{1}{n_0_{\max}} \right) - \log \left( \frac{1}{n_0_{\min}} \right)}
\]

7. Sub variables

In this paper, a main variable c is considered to be a function of load level and material property (crack propagation rate). For example, Equation (21) is assumed:

\[
c = \left( \frac{S}{S_0} \right)^{b_1} \left( \frac{c_m}{c_{m,0}} \right)^{b_2}
\]

Where, S is load and c_m is a constant that represents crack propagation rate. Subscript 0 represents a standard value.

8. Determination of influence coefficient vector and total quality assurance index

Based on the above results, an influence coefficient vector can be determined by Eq. (22)

\[
\alpha q_1 + \beta q_2 + \gamma q_3 = \delta
\]

\[
\alpha^2 + \beta^2 + \gamma^2 = 1
\]

\[
C = \begin{pmatrix}
\alpha \\
\beta \\
\gamma
\end{pmatrix}
\]

Influence coefficients thus determined is shown in Table 1. Influence coefficients depend on the cumulated failure probability (target failure probability), however, the change is of the influence coefficients is not significant.

The total quality assurance index F can be determined by Eq. (24).

\[
F = \delta
\]

In this case, a liner relationship is observed between F and the logarithm of failure probability. This is shown in Fig. 5.

As a result of the above discussions, Eq.(9) can be written as Eq. (25) for this specific example problem.
IV. Example of margin exchange evaluation based on the Vector Method

1. Procedure of margin exchange evaluation

A procedure of margin exchange evaluation based on the Vector Method is shown below:

1. Determine main and sub design variables and formulate failure probability as a function of these variables.

2. Based on the formulation, using either analytical or numerical methods, determine combinations of variables that give an identical failure probability, i.e., an equi-failure surface $G$. Repeat this procedure for several levels of failure probability.

3. Determine an appropriate projection so that $G$ is projected to an equi-failure probability plane $H$. When $G$ is expressed as a product of power of main design variables, the logarithm of $G$ gives $H$.

4. Determine an influence coefficient vector as a normal vector of the plane $H$. Determine the total quality assurance index as the distance between the origin and the plane $H$.

5. Tabulate influence coefficient vector and the relationship between quality assurance indices and design variables. This is a “margin exchange table”.

6. Using the margin exchange table, find a combination of quality assurance indices that satisfy the total quality assurance index corresponding to target reliability. If more than one combinations satisfy the total quality assurance index, then margin exchange holds among those combinations of design variables.

2. Example of a margin exchange table

An example of a margin exchange table is shown in Table 2. This example expresses a detailed method that gives influence coefficients as a function of target reliability.

V. Discussions

The procedure requires as its first step that failure probability be calculated for as many cases as sufficient to determine equi-failure probability surfaces. For this, purpose, effective means of calculation is necessary. Although the algebraic method proposed in the study is quite effective for particular conditions, applicability is limited compared to numerical methods. The expansion of the algebraic method and the speeding up of numerical methods are necessary.

In the example of this paper, the logarithm of an equi-failure probability surface $G$ gave an equi-failure probability. This observation may be applied to cases where failure probability is a function of crack propagation rate in a structure, ISI accuracy, and ISI frequency.

In this paper, failure probabilities at 1000 cycles were examined. In general, failure surfaces, i.e., influence coefficients depend on a number of cycles. A sensitivity study is necessary.

In this method, the relationship between failure probability and a total quality assurance index depend on the definition range of design variables. The range used in practical design must be examined.

VI. Conclusions

The conclusions obtained in this paper are as follows:

(1) For use in margin exchange evaluation, a new method, a “Vector Method” was proposed. This method enables easy and quick margin exchange evaluation based on equi-failure probability surfaces, which are to be obtained beforehand.

(2) This method gives a probabilistic background to the quality assurance index proposed in the study of the system based code.

(3) In an example calculation, accuracy of margin exchange evaluation was an order of magnitude in terms of failure probability.

(4) For more efficient calculation of failure probability, an algebraic method was proposed. This method can be applied in particular conditions in place of numerical methods such as Monte-Carlo simulation, which tend to need certain computation time.

References

3) Asayama, T., Morishita, M., Dozaki, K. and Higuchi, M., Development of the System Based Code for Fast Breeder Reactors and Light Water Reactors - Basic scheme -, ICONE10
Table 1  Influence coefficient and total quality assurance index

<table>
<thead>
<tr>
<th>Target reliability (n=1000)</th>
<th>Influence coefficient</th>
<th>Total quality assurance index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load/Material (c1)</td>
<td>ISI(Accuracy) (c2)</td>
<td>ISI(Frequency) (c3)</td>
</tr>
<tr>
<td>1.0E-03</td>
<td>0.488</td>
<td>0.393</td>
</tr>
<tr>
<td>1.0E-04</td>
<td>0.455</td>
<td>0.446</td>
</tr>
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<td>1.0E-05</td>
<td>0.440</td>
<td>0.457</td>
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<tr>
<td>1.0E-06</td>
<td>0.447</td>
<td>0.468</td>
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<td>1.0E-07</td>
<td>0.425</td>
<td>0.555</td>
</tr>
<tr>
<td>Average</td>
<td>0.451</td>
<td>0.464</td>
</tr>
</tbody>
</table>

Table 2  An example of a margin exchange table

\[
F = c_1 q_1 + c_2 q_2 + c_3 q_3 \quad \ldots (1) \\
q_i = c_{i1} q_{i1} + c_{i2} q_{i2} + c_{i3} \quad (c_{i3} = -3.447) \quad \ldots (2)
\]

<table>
<thead>
<tr>
<th>Partial code (i)</th>
<th>Technical item</th>
<th>Target failure probability (P_f)</th>
<th>Total quality assurance index (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (11)</td>
<td></td>
<td>(1 \times 10^{-3})</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1 \times 10^{-4})</td>
<td>5.4</td>
</tr>
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<td></td>
<td>(1 \times 10^{-5})</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1 \times 10^{-6})</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1 \times 10^{-7})</td>
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</table>

Main influence coefficient

<table>
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<tr>
<th>Load/Material (1)</th>
<th>c_{i1}=0.451</th>
<th>c_{i1}=1.289</th>
<th>q_{i1}</th>
</tr>
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<tbody>
<tr>
<td>(1 \times 10^{-3})</td>
<td>0.488</td>
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<tr>
<td>(1 \times 10^{-4})</td>
<td>0.455</td>
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<tr>
<td>(1 \times 10^{-7})</td>
<td>0.425</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Material (12)  | c_{i2}=0.451 | c_{i2}=1.230 | q_{i2} |
<table>
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<td>2</td>
</tr>
<tr>
<td>(1 \times 10^{-7})</td>
<td>0.425</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Inservice Inspection (ISI)  | c_{i3}=0.464 |              | q_{i3} |
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<tbody>
<tr>
<td>(1 \times 10^{-3})</td>
<td>0.393</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>(1 \times 10^{-4})</td>
<td>0.446</td>
<td></td>
<td>4</td>
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<tr>
<td>(1 \times 10^{-5})</td>
<td>0.457</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>(1 \times 10^{-6})</td>
<td>0.468</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>(1 \times 10^{-7})</td>
<td>0.555</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Inspection frequency (f)  | c_{i3}=0.760 | -            | q_{i3} |
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<td>(1 \times 10^{-3})</td>
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<td>(1 \times 10^{-4})</td>
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<td>(1 \times 10^{-6})</td>
<td>0.762</td>
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<td>2</td>
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<tr>
<td>(1 \times 10^{-7})</td>
<td>0.715</td>
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</table>

*(1) Ratio to nominal load level determined elsewhere.
*(2) Ratio to nominal crack propagation rate determined elsewhere.
*(3) Average value independent of crack depth.
Fig. 1  Fishbone diagram for an example problem

\[ x_1: \text{Crack propagation rate in structure} \]
\[ x_1 = \frac{da}{dn} = ca^m \]
\[ x_{11}: \text{Load level} \]
\[ x_{12}: \text{Material property} \]
\[ x_2: \text{ISI frequency (POD)} \]
\[ x_3: \text{ISI Frequency} \]
\[ c = c(x_{11}, x_{12}) \]

Fig. 2  Equi-failure probability surface \( X (P_f=1\times10^{-3}) \)
Fig. 3  Equi-failure probability surface $X \left( P_f=1\times10^{-3} \right)$

Fig. 4  Equi-failure probability surface $X \left( P_f=1\times10^{-3} \right)$
Fig. 5  Relationship between cumulated failure probability and total quality assurance index